

Summary

- Based on preliminary research, pre-2020 calibration criteria and AIRG returns appear to be a reasonable reference point, and improving the alignment of cumulative wealth factors may improve the interpretability of field test results.
 - Updated acceptance criteria should be developed concurrently to the field test.
- Comparing GEMS equity calibrations to AAA returns using cumulative wealth factors:
 - Significant increase in severity of down and up outcomes over the long term (see slide 3)
 - Even with use of jump diffusion, GEMS does not produce more severe tails over the short term (see slide 5)
- Historical calibration suggests higher mean reversion of variance, and lower frequency of jumps, both of which would help mitigate extreme outcomes over the long term.
 - Higher mean reversion of variance is present regardless of modeled jumps process
 - Reducing the frequency of jumps has meaningful improvement to MLE log-likelihood, and suggests more of the monthly equity return can be explained using stochastic variance process
 - Proposed ACLI GEMS calibration would produce more reasonable distribution and align with current Academy scenarios (as well as other standard models calibrated to history) in the low tails
- Rate/ERP relationship in GEMS drives substantial impact on long term wealth factors especially in high tail scenarios.
 - See Slide 4 for details.
 - Further modifications to modeled ERP/rate relationship, such as targeting constant return/inverse rate-ERP might help produce more reasonable and stable distribution of outcomes but are not in scope for this alternative GEMS calibration.

Calibration and Parameters

	Calibrated to monthly data 3/1957-12/2020							
	GEMS A	GEMS G	GEMS H	No Jumps	GEMS Jumps	GEMS Jump Size	Adjusted Mu1	ACLI Alternative
mu0	0.05193	0.06507	0.05312	0.03637	0.02022	0.03669	0.02945	0.04040
mu1	0.09257	0.09708	1.09708	0.06825	0.00000	0.06097	1.50000	1.50000
alpha	0.00556	0.00586	0.00586	0.02118	0.01419	0.02130	0.02192	0.02192
beta	0.39711	0.49081	0.49081	1.01197	1.02388	1.11078	1.22990	1.22990
sigma	0.08187	0.07160	0.07160	0.13505	0.08896	0.12709	0.12478	0.12478
ret/var correl.	-0.48000	-0.48000	-0.48000	-0.58194	-0.60011	-0.61919	-0.63185	-0.63185
mu_jump	-0.05558	-0.05558	-0.05558	0.00000	-0.05558	-0.05558	-0.05558	-0.05558
sigma_jump	0.05750	0.05750	0.05750	0.00000	0.05750	0.05750	0.05750	0.05750
lambda_jump	139.58820	139.58820	139.58820	0.00000	139.58820	15.05324	15.10181	15.10181
initial vol	0.09434	0.10909	0.10909	0.14469	0.11772	0.13847	0.13351	0.13351
Log Likelihood	1375	1378	1377	1389	1382	1394	1393	

ACLI Alternative GEMS calibration considerations:

- Model parameters calibrated to historical data
- mu1 risk premium coefficient was set to 1.5, as a calibration constraint, to help mitigate long term tails
- Jump size parameters (mu_jump and sigma_jump) were set equal to GEMS original
- mu0 parameter was set to target 9.5% mean cumulative wealth ratio over 30 years
 - More severe than historical data
 - Similar to GEMS G/H sensitivities

- GEMS model parameters were calibrated to historical monthly ERP (S&P price return less monthly return on 3m Treasury rate) from 3/1957 to 12/2020, using generalized MLE.
- Historical calibration points to:
 - Higher mean reversion of variance, or higher *beta* parameter
 - Lower frequency of jumps – lower *lambda_jump* parameter

Distribution of Cumulative Wealth Factors

Conning w/ GFF 12.31.20 Wealth Factors (H)						
	12	60	120	240	360	600
S&P 500	1 Yr	5 Yr	10 Yr	20 Yr	30 Yr	50 Yr
Min	0.44	0.19	0.20	0.24	0.21	0.32
1.0%	0.70	0.54	0.52	0.65	0.90	2.12
2.5%	0.76	0.65	0.68	0.90	1.34	3.74
5.0%	0.81	0.75	0.83	1.15	1.85	5.60
10.0%	0.87	0.87	1.02	1.59	2.67	9.13
25.0%	0.97	1.11	1.42	2.52	4.79	19.39
50.0%	1.07	1.39	1.98	4.20	9.20	48.16
75.0%	1.17	1.72	2.68	6.79	17.57	122.47
90.0%	1.26	2.02	3.48	10.37	31.47	294.37
95.0%	1.32	2.23	4.10	13.29	45.61	527.96
97.5%	1.36	2.41	4.68	17.05	62.32	853.21
99.0%	1.41	2.67	5.58	23.01	92.93	1,718.15
Max	1.65	4.41	14.58	67.33	588.35	26,117.11

ACLI Alternative w GFF 1m Rates						
	12	60	120	240	360	600
S&P 500	1 Yr	5 Yr	10 Yr	20 Yr	30 Yr	50 Yr
Min	0.44	0.29	0.25	0.32	0.26	0.49
1.0%	0.72	0.60	0.64	0.82	1.25	3.43
2.5%	0.78	0.71	0.77	1.09	1.74	5.22
5.0%	0.83	0.81	0.92	1.39	2.35	7.58
10.0%	0.89	0.92	1.12	1.81	3.20	11.57
25.0%	0.98	1.14	1.50	2.75	5.34	23.09
50.0%	1.07	1.40	2.00	4.31	9.63	52.76
75.0%	1.17	1.69	2.65	6.76	17.46	125.59
90.0%	1.25	1.98	3.36	10.13	30.83	299.21
95.0%	1.30	2.16	3.93	12.98	45.25	531.09
97.5%	1.34	2.32	4.42	16.81	63.17	887.59
99.0%	1.39	2.55	5.07	22.04	96.36	1,620.42
Max	1.59	3.34	9.59	70.80	1,066.70	16,847.51

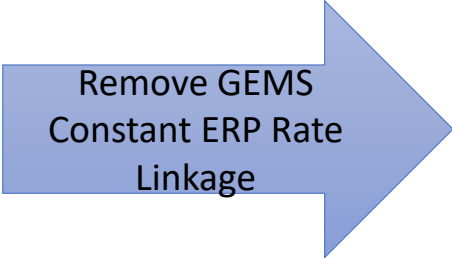
AAA Prepackaged CSV (Dec. 2005)					
	12	60	120	240	360
S&P 500	1 Yr	5 Yr	10 Yr	20 Yr	30 Yr
Min	0.41	0.32	0.26	0.35	0.38
1.0%	0.70	0.62	0.66	0.83	1.22
2.5%	0.76	0.72	0.77	1.10	1.69
5.0%	0.82	0.81	0.92	1.41	2.25
10.0%	0.89	0.93	1.12	1.83	3.09
25.0%	0.98	1.16	1.51	2.74	5.11
50.0%	1.09	1.45	2.09	4.27	8.84
75.0%	1.19	1.81	2.88	6.80	15.35
90.0%	1.30	2.22	3.81	10.15	24.98
95.0%	1.37	2.48	4.44	12.92	34.25
97.5%	1.44	2.72	5.17	15.65	45.88
99.0%	1.52	3.06	6.18	20.49	60.45
Max	1.92	4.77	11.86	66.94	235.95

- ACLI Alternative Calibration produces low tails that are closer to AIRG, due to higher mean reversion of variance and lower frequency of jumps
- Both, GEMS H, and ACLI wealth factors include GFF treasury rates as portion of the return, and were modeled under best-efforts interpretation of GEMS equity model
- Much higher wealth factors in upper tails vs AIRG are due to the treasury rate component of the return

GEMS Constant ERP Rate Linkage Widens Long Term Wealth Factor Tails

WF30: GEMS ERP/Rates		
S&P 500	GEMS H	ACLI Alternative
Min	0.21	0.26
1.0%	0.90	1.25
2.5%	1.34	1.74
5.0%	1.85	2.35
10.0%	2.67	3.20
25.0%	4.79	5.34
50.0%	9.20	9.63
75.0%	17.57	17.46
90.0%	31.47	30.83
95.0%	45.61	45.25
97.5%	62.32	63.17
99.0%	92.93	96.36
Max	588.35	1,066.70
average	9.43%	9.50%

Remove GEMS
Constant ERP Rate
Linkage

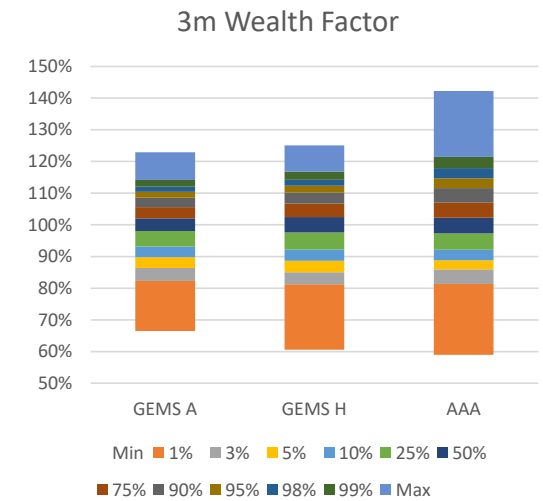


WF30: Constant Return			
S&P 500	GEMS H	ACLI Alternative	AAA
Min	0.27	0.33	0.38
1.0%	1.06	1.62	1.22
2.5%	1.67	2.16	1.69
5.0%	2.23	2.82	2.25
10.0%	3.12	3.74	3.09
25.0%	5.32	6.00	5.11
50.0%	9.47	9.92	8.84
75.0%	16.02	16.01	15.35
90.0%	25.09	23.98	24.98
95.0%	32.43	29.94	34.25
97.5%	40.72	36.44	45.88
99.0%	53.51	46.33	60.45
Max	153.26	103.23	235.95
average	8.75%	8.75%	8.80%

* Return drift adjusted to match 8.75% average wealth factor

Short Term Equity Returns

	3M Return				6M Return				12M Return		
	GEMS A	GEMS H	AAA		GEMS A	GEMS H	AAA		GEMS A	GEMS H	AAA
Min	-33%	-39%	-41%	Min	-40%	-40%	-53%	Min	-53%	-54%	-60%
1%	-18%	-19%	-19%	1%	-22%	-24%	-24%	1%	-29%	-30%	-28%
3%	-14%	-15%	-14%	3%	-18%	-19%	-19%	3%	-24%	-24%	-22%
5%	-10%	-11%	-11%	5%	-14%	-15%	-14%	5%	-18%	-19%	-17%
10%	-7%	-8%	-8%	10%	-10%	-10%	-10%	10%	-12%	-13%	-12%
25%	-2%	-2%	-3%	25%	-3%	-3%	-3%	25%	-3%	-3%	-2%
50%	2%	2%	2%	50%	4%	4%	4%	50%	6%	7%	8%
75%	6%	7%	7%	75%	9%	11%	11%	75%	14%	17%	19%
90%	9%	10%	12%	90%	13%	16%	18%	90%	22%	26%	30%
95%	10%	12%	15%	95%	16%	19%	23%	95%	26%	31%	36%
98%	12%	14%	18%	98%	19%	22%	27%	98%	31%	35%	42%
99%	14%	17%	21%	99%	21%	25%	33%	99%	35%	41%	52%
Max	23%	25%	42%	Max	38%	42%	71%	Max	69%	75%	85%



- Academy equity returns are more disperse and produce more severe high and low wealth factors compared to GEMS
- GEMS use of jump diffusion under proposed parameterization does not translate into more sever tails over the short term