ACLI
Financial Security for Life

ACLI Dynamic Generalized Fractional Floor (GFF) Proposal

October 2, 2024

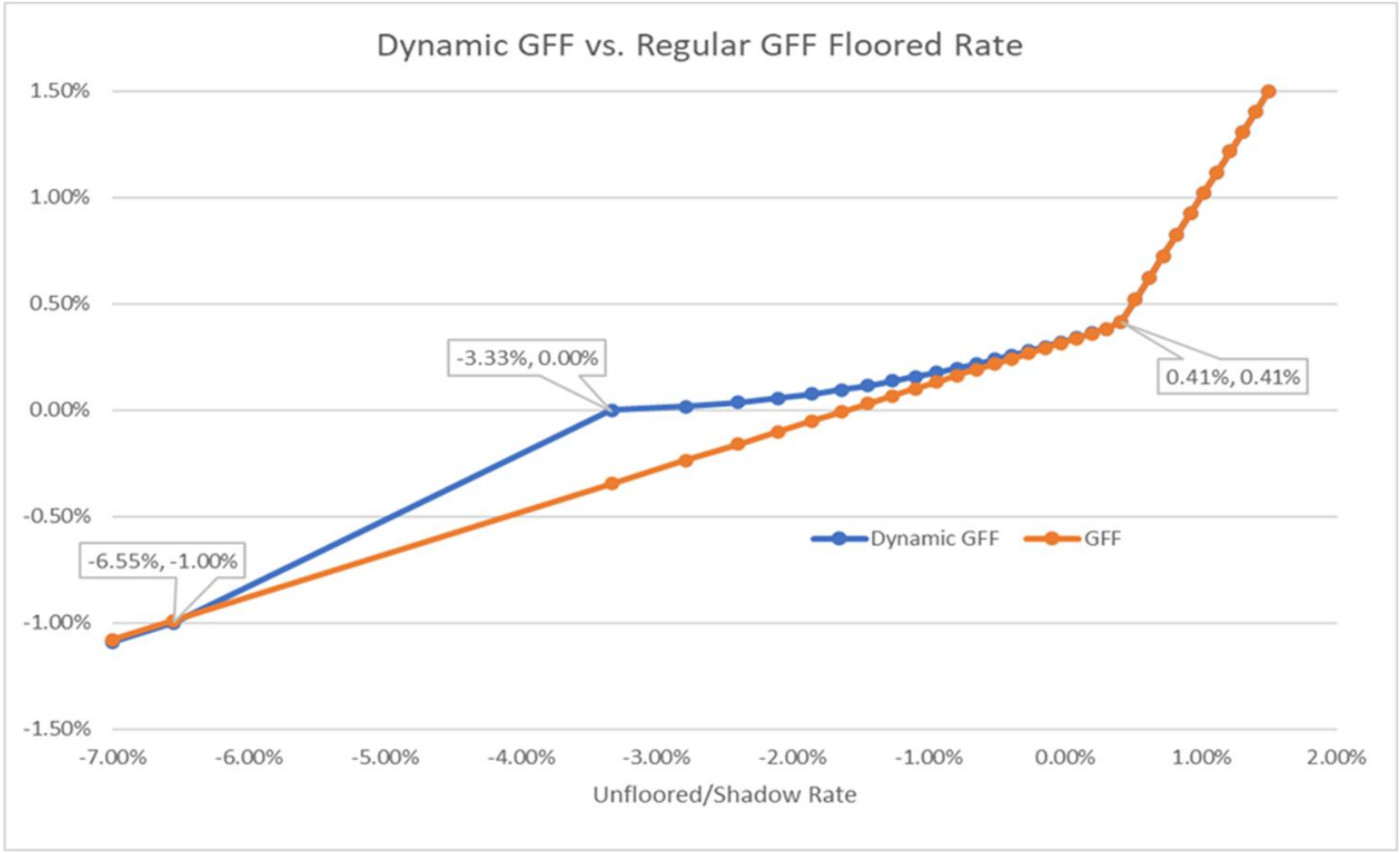
Generalized Fractional Floor (GFF)

- In 2021, discussions began to introduce a flooring mechanism to the Generator.
- The 3-factor CIR model, coupled with the low-for-long criteria, can be challenged to simultaneously produce high historical rates (1980s) without producing extremely negative and volatile short rates.
- To address this challenge, several floor proposals were discussed, ultimately resulting in reflecting a GFF in the Generator.
- While the GFF produced some improvements, the frequency of negatives is still quite severe.

Dynamic GFF

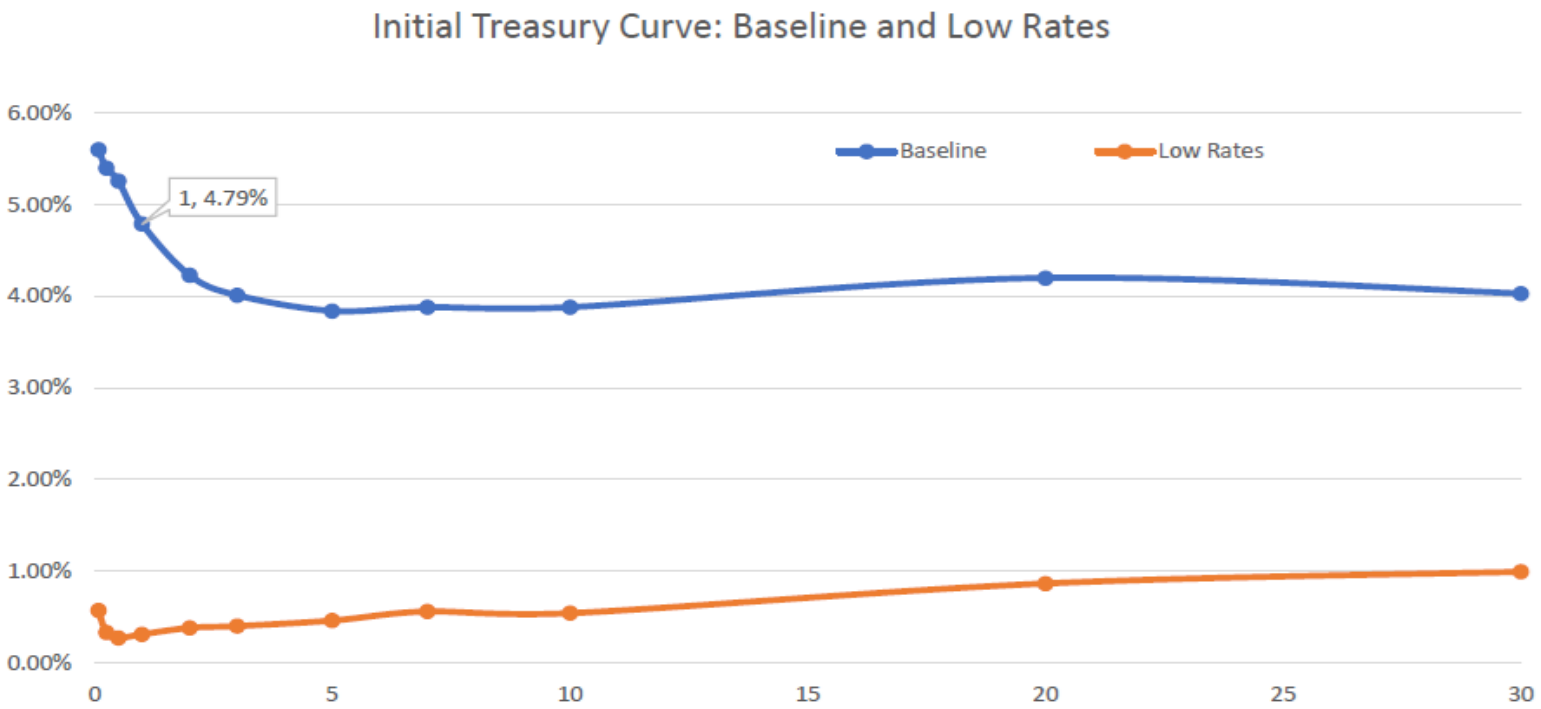
- ACLI is proposing a dynamic GFF to provide flexibility to more precisely calibrate the appropriate level of modeled negative rates in terms of both frequency and severity.
- In addition to the Current GFF terms, would need to define:
  - Minimum floored rate (-1% which aligns with the current GOES scenarios and targeting criteria);
  - The desired frequency of the negative rates in the steady state distribution (1% which aligns with the Academy worse than history criteria)
- Once these are defined, the floor formula parameters would be set based on how these two criteria interact with the relevant unfloored (shadow) rate distribution

Resulting steady-state (years 80+) floored rates as of 12/31/2023 (FT2 Baseline):

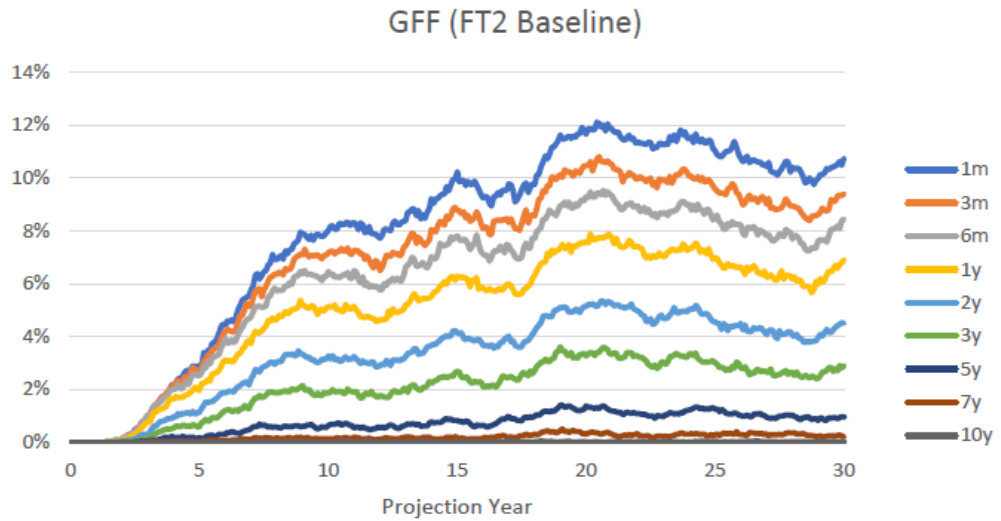


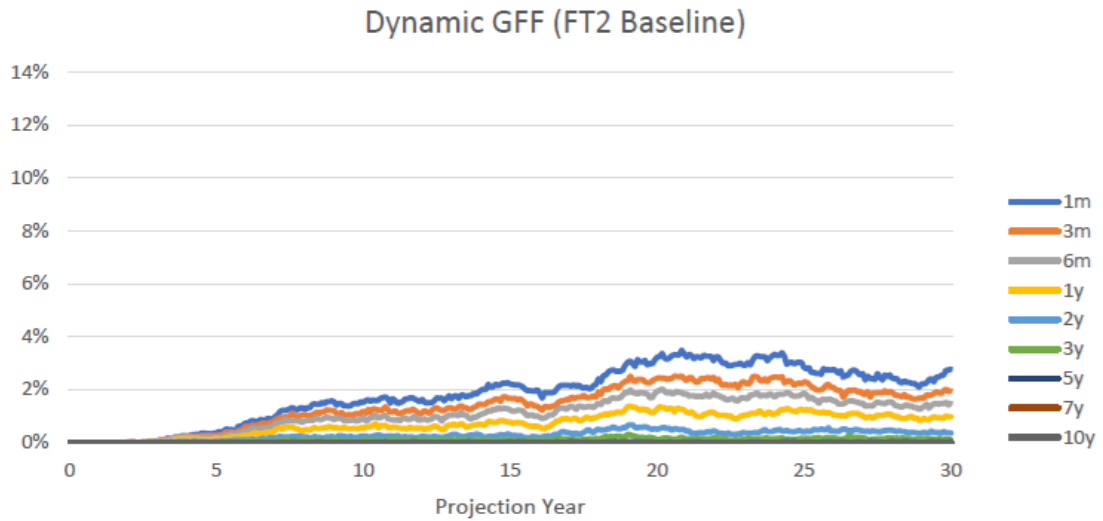
Initial Treasury Curve: Baseline (12/2023) and Low Rates (3/2020)

- Distribution of negative rates is sensitive to starting conditions
- The next slides compare negative rate distributions between current and dynamic GFF on these valuation dates used in FT2



Frequency of Negative Rates: Baseline FT2 Scenarios (first 30 years of the projection)



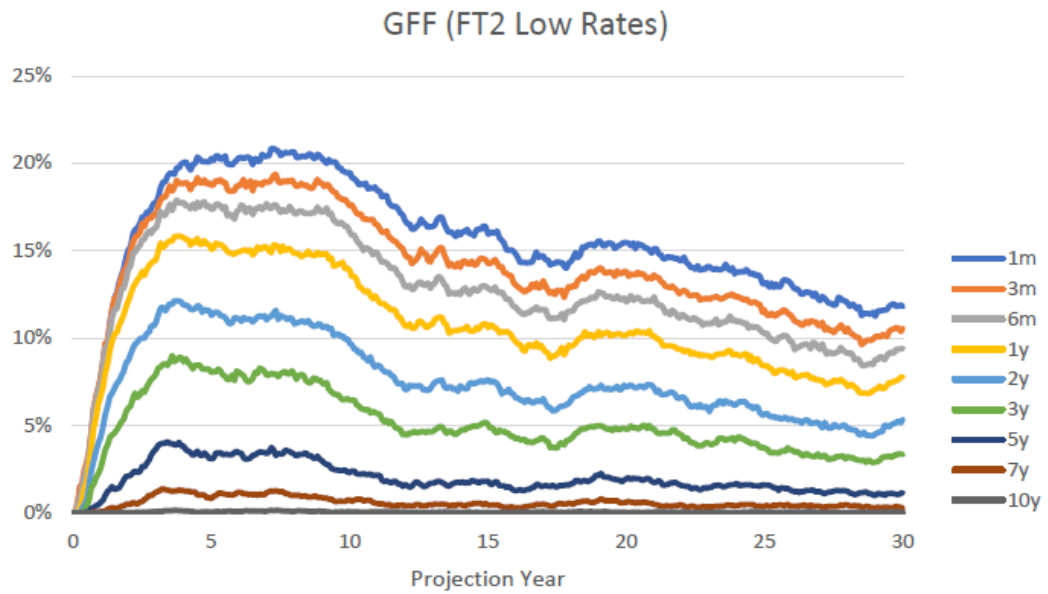


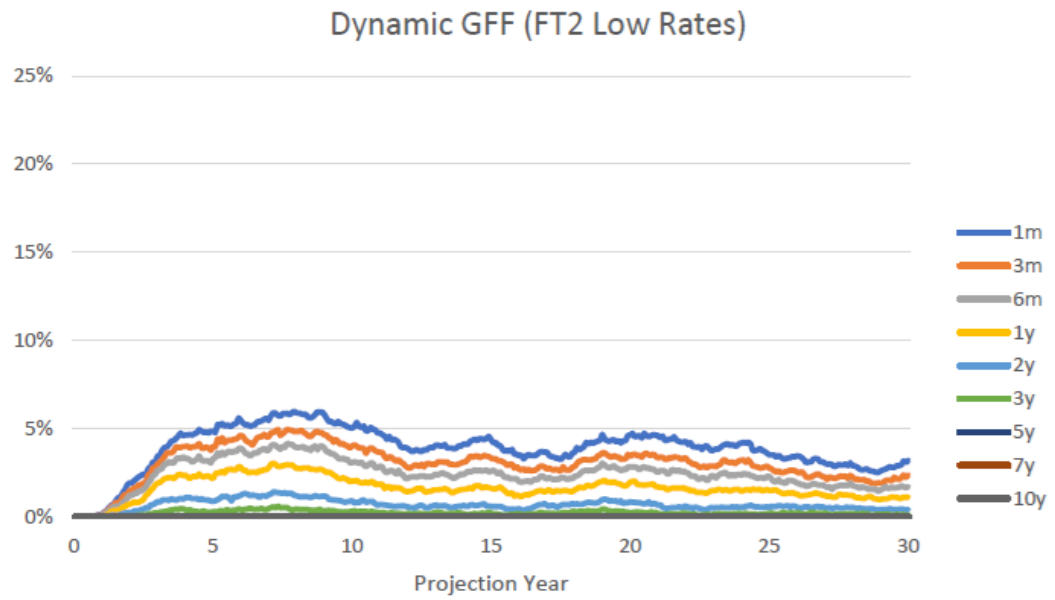
Frequency of Negative 1yr Rate by Projection Year

Proj. Year	GFF	Dynamic GFF
1	0.0%	0.0%
2	0.1%	0.0%
5	2.0%	0.2%
10	5.1%	0.5%
20	7.6%	1.3%
30	6.9%	1.0%

- Under current GFF up to 12% of 1-month rates, and 8% of 1-year rates fall below 0% in the Baseline scenarios
- Dynamic GFF significantly mitigates frequency of negative rates in the projection.
- Frequency of negative rates is diminished in the early years due to high starting rates (UST 1yr = 4.79%)

Frequency of Negative Rates: Low Rates FT2 Scenarios ((first 30 years of the projection)





Frequency of Negative 1yr Rate by Projection Year

Proj. Year	GFF	Dynamic GFF
1	6.7%	0.1%
2	12.1%	0.7%
5	15.1%	2.2%
10	13.8%	2.1%
20	10.2%	2.0%
30	7.8%	1.1%

- Current GFF produces up to 20% of 1-month rates, and 15% of 1-year rates below 0% in the Low Rate scenarios
- Dynamic GFF significantly mitigates frequency of negative rates in the projection. Note that the frequency of negative rates is higher compared to the Baseline scenario set under both flooring methods, since the initial curve is 3%+ lower.

Appendix

Current Generalized Fractional Floor (GFF)

$$rate(s) = max(\kappa + m\ (s - \kappa), s)$$

- Where:

  $\boldsymbol{s}$ is the natively modeled shadow, or unfloored, rate

  $\boldsymbol{rate(s)}$ is the floored rate as a function of the shadow rate s and the GFF parameters κ and *m*

  $\boldsymbol{\kappa}$ is the threshold parameter – shadow rates below this threshold are subject to the fractional flooring

  $\boldsymbol{m}$ is the constant fraction parameter which applies to the difference $s - \kappa$. Setting *m=0* would imply simple flooring at k, while *m=1* would imply no flooring as *rate(s) = s*

- For purposes of GOES, GFF parameters are set to: $\kappa = .004$ and $m = .2$, and the floor applies to the continuous spot rates generated by a 3-factor CIR model

Proposed Dynamic Generalized Fractional Floor (GFF)

$$rate(s) = max(\kappa + m(s)\ (s - \kappa), s)$$

$$m(s) = m_0 + max(min(s, \kappa) - s_0, 0)\ R_0 - max(s_0 - max(s, s_{min}), 0)\ R_{min}$$

- Where:

  $\bar{m}$ is the terminal fraction level that applies when $s = \kappa$; subject to constraint $\bar{m} < \frac{2\kappa}{\kappa - s_0}$

  $m_0 = \frac{\kappa}{\kappa - s_0}$ is the fraction that ensures $rate(s_0) = 0$

  $R_0 = \frac{\bar{m} - m_0}{\kappa - s_0}$

  $m_{min} = \frac{\kappa - rate_{min}}{\kappa - s_{min}}$ is the fraction that ensures $rate(s_{min}) = rate_{min}$

  $R_{min} = \frac{m_0 - m_{min}}{s_0 - s_{min}}$

- We assume m(s) can be recast as a piecewise linear function, based on additional targets to explicitly control for (i) frequency of negative rates and (ii) minimum floored rate boundary

Recipe for Setting Dynamic GFF Parameters

1. Start with the core GFF parameters, $\kappa = .004$ and $\bar{m} = .2$
2. Produce the target distribution of shadow rates as basis for targeting: include tail percentile levels such as minimum, 1%, 2%, etc. and pick the desired short rate tenor, such as 1yr.
3. Target negative frequency: s_0 = -3.3% which is the 1st percentile of the 1yr shadow rate distribution in years 81-100 (steady state) FT2 baseline scenarios. Note that this could also be set to 1.5% or 2% tail levels, to allow for more negative rates in the distribution.
4. Check to see if $\bar{m}$ satisfies the constraint (which it does), and lower accordingly.
5. Set the low-rate boundary (the ultimate floor): $s_{min} = -6.55\%$ (minimum shadow rate in FT2 scenarios) and $rate_{min} = -1\%$. Any other suitable level, like -1.5% would also work. Note that the FT2 baseline scenario 1yr spot rate bottoms out at ~ -1% as well.

This parameterization results in the following dynamic fraction $m(s)$;
The fraction $m(s)$ linearly grades from $\bar{m}$ to m_0 at s=-.0333, to m_{min} at s = .0655 as intended.