PINNACLE
ACTUARIAL RESOURCES

July 23, 2024

STEPPING INTO THE ACTUARIAL MODELING WONDERLAND

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Agenda

1. Paradox Overview
2. Simpson's Paradox
3. Sleeping Beauty Paradox
4. Berkson's Paradox

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PARADOX OVERVIEW

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Mathematical Paradoxes

> *"A mathematical paradox is a **mathematical conclusion** so **unexpected** that it is difficult to accept even though every step in the **reasoning is valid**."*

- Britannica

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SIMPSON'S PARADOX

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Simpson’s Paradox

Simpson’s Paradox is a statistical phenomenon where an **association between two variables in a population emerges, disappears or reverses when the population is divided into subpopulations**.

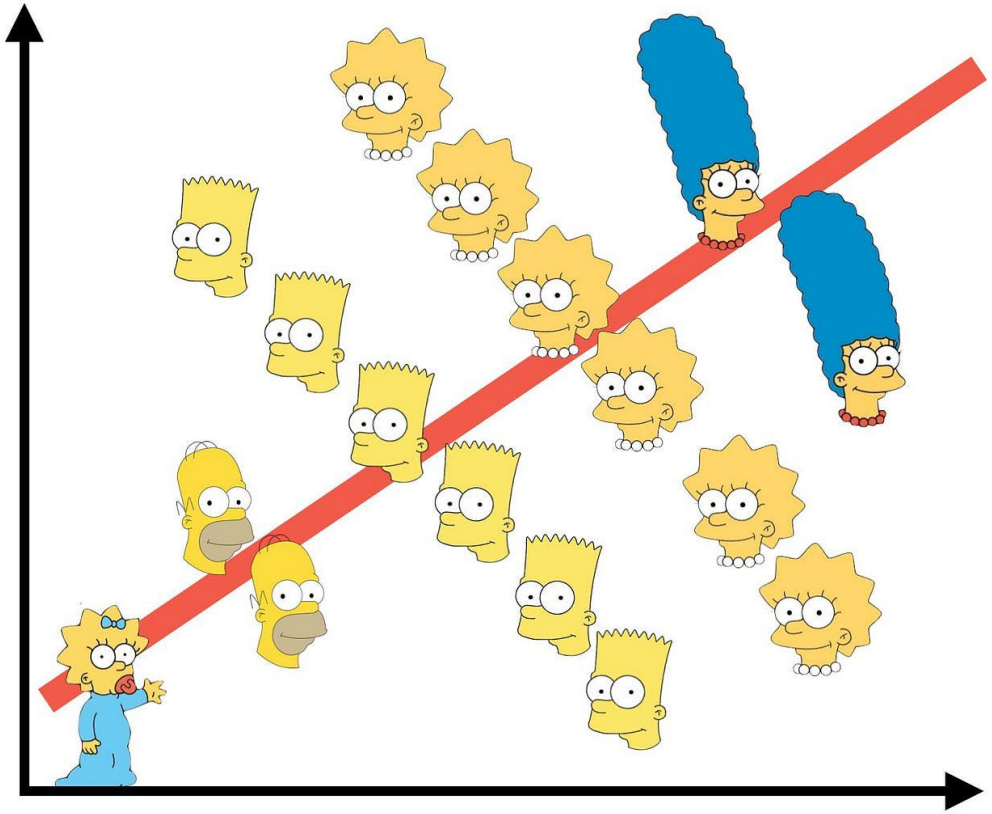
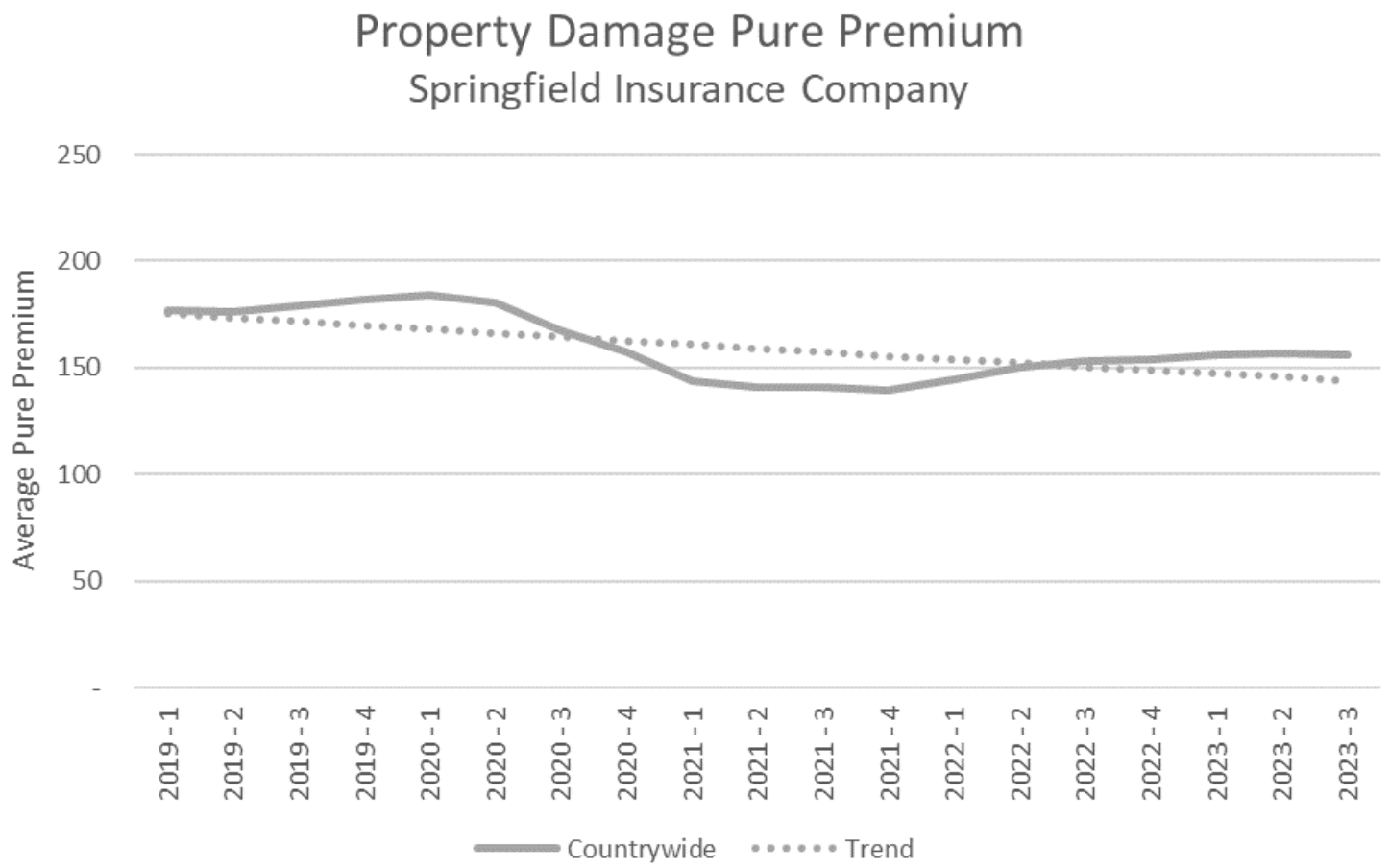


Image: Medium.com; Analytics Vidhya

Simpson's: Insurance Example

Springfield Insurance Company is analyzing loss trend on their historical private passenger auto book.

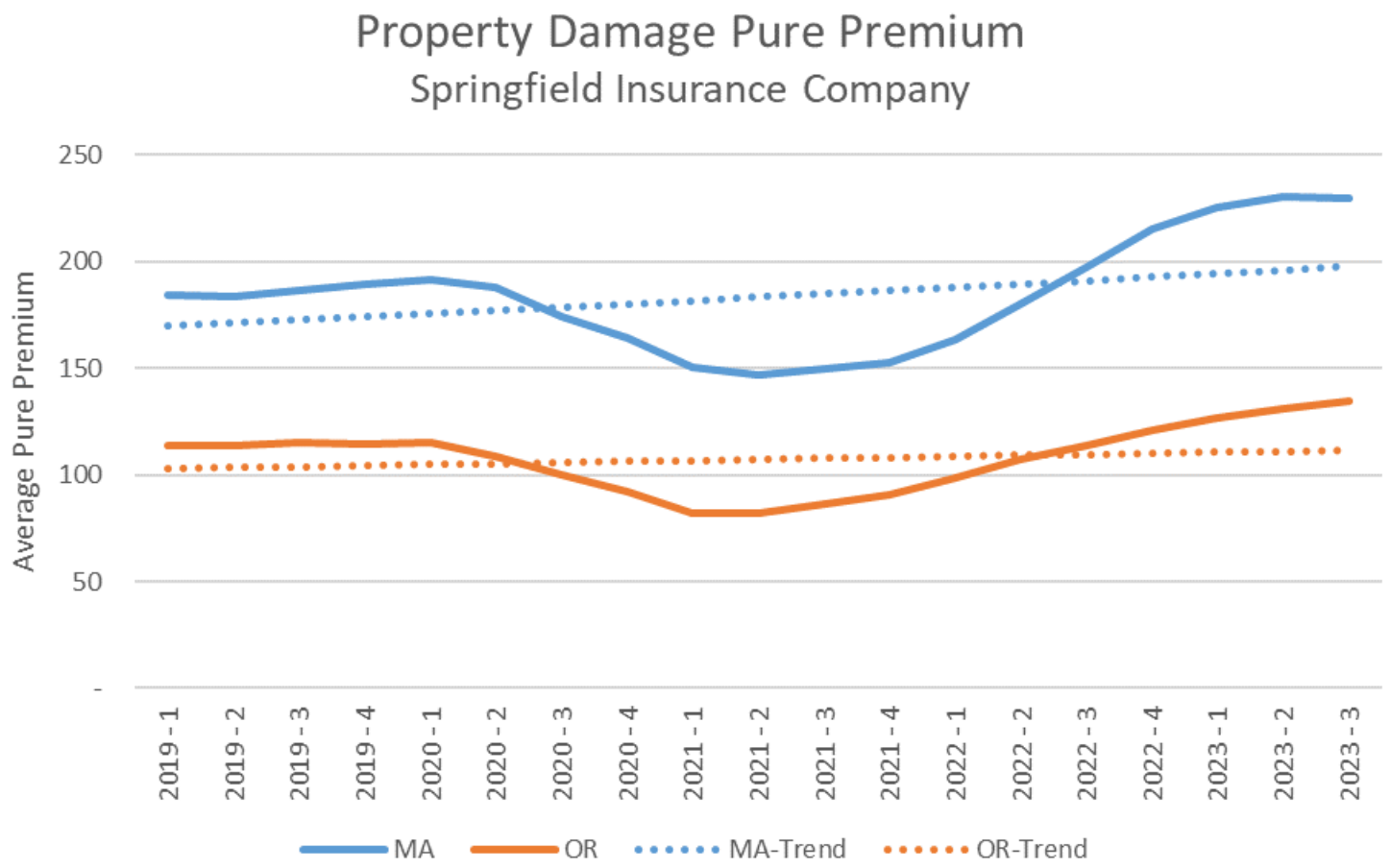
OVERALL 5-YR TREND: -1.1%



Simpson's: Insurance Example

5-YR PURE PREMIUM TREND

- **OVERALL: -1.1%**
- **Massachusetts: +0.8%**
- **Oregon: +0.4%**



Simpson's: Insurance Example

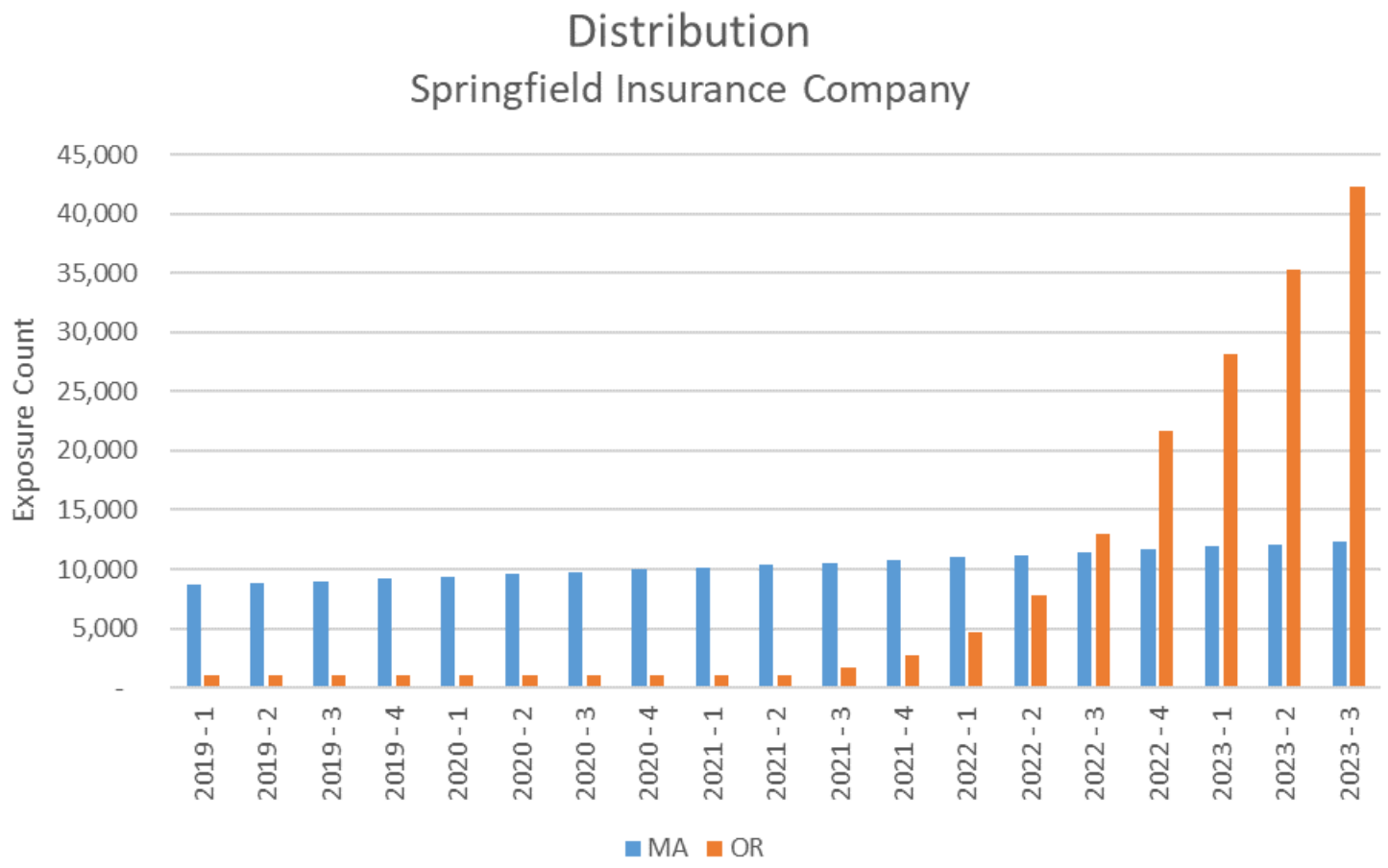
DISTRIBUTIONAL SHIFT BY STATE

Massachusetts

- Started writing prior to 2019
- Quarterly growth of ≈2%

Oregon

- Low exposure amount
- Exposure ramped up quickly in late 2021

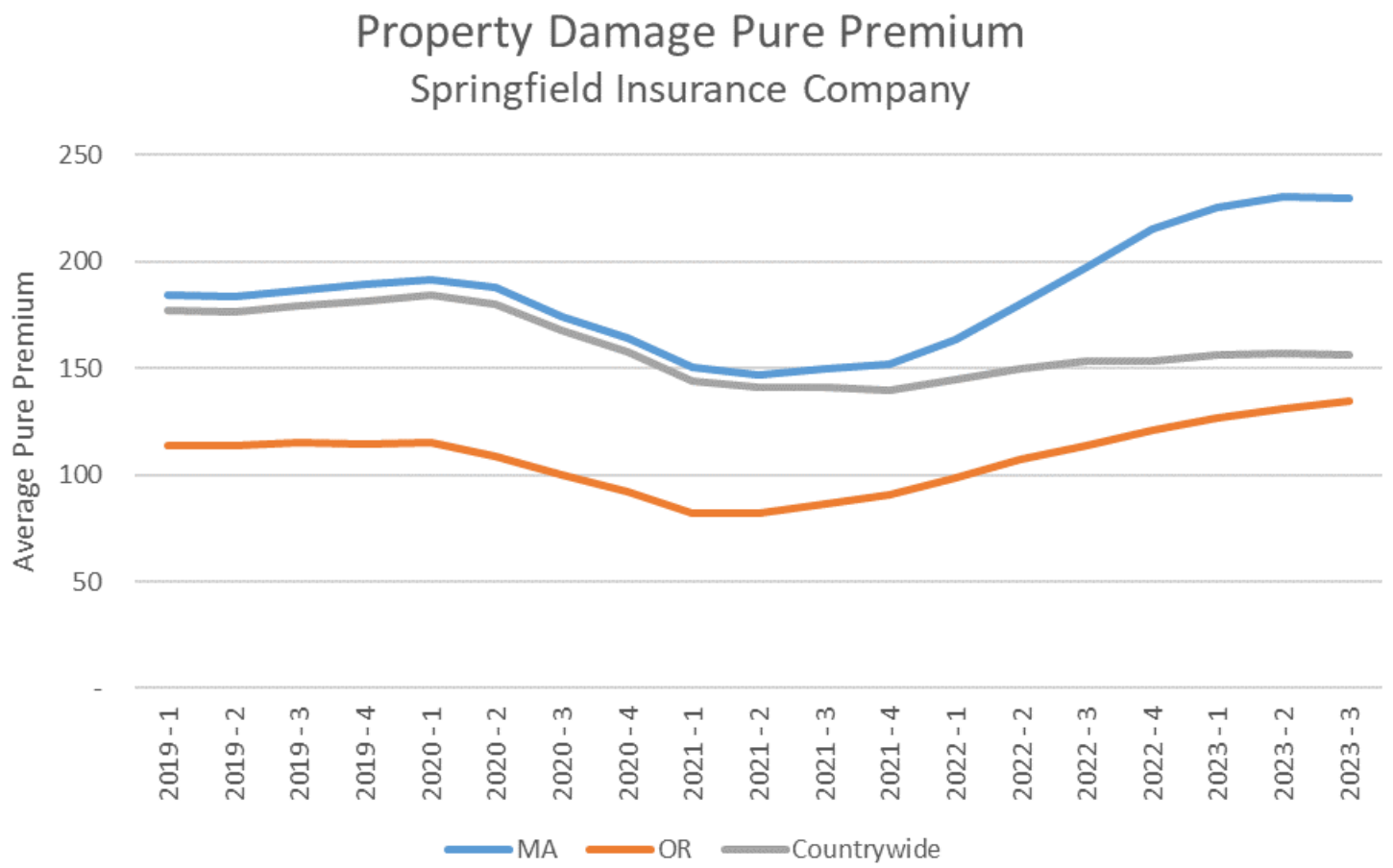


Simpson's: Insurance Example

5-YR AVG PURE PREMIUM

Massachusetts: $188

Oregon: $125



Simpson's Takeaways

- Recognize that overall result might not be the total story
- Decompose the effects
- Evaluate data on a normalized basis

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SLEEPING BEAUTY PARADOX

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Sleeping Beauty Paradox

The Sleeping Beauty Paradox is a decision theory puzzle that deals with the logic and uncertainty of experience, or the idea that **perspective shapes rational conclusions**.

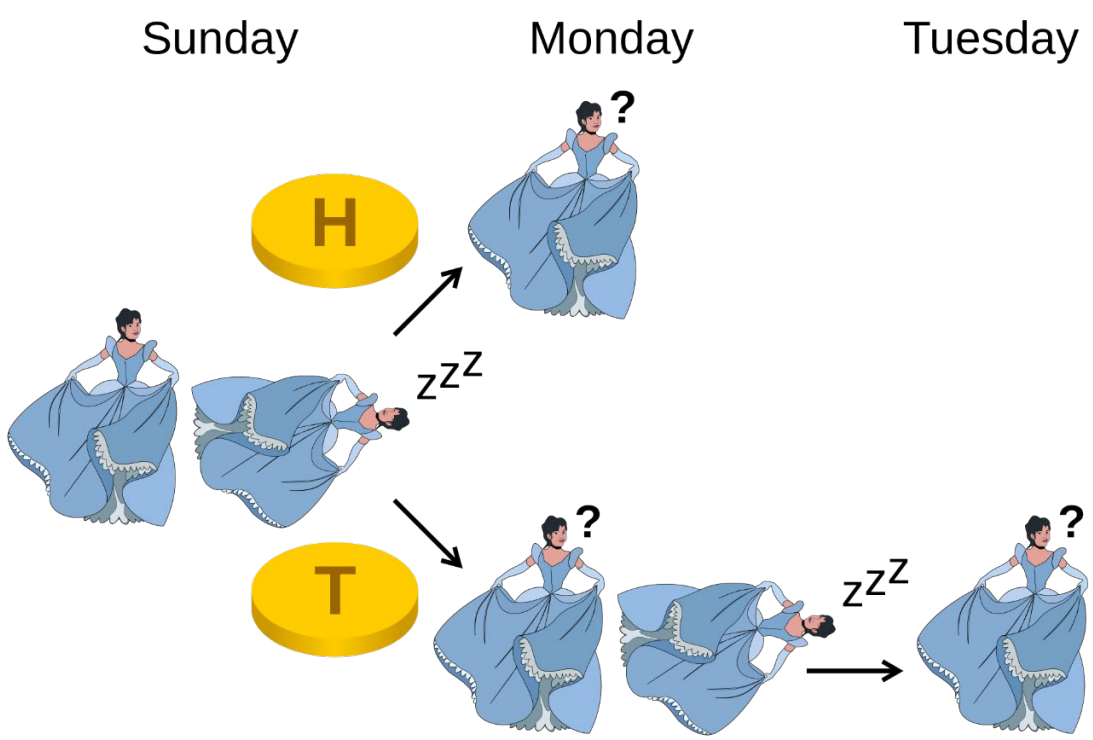

Image: Wikipedia.org; Sleeping Beauty Problem

Sleeping Beauty Paradox

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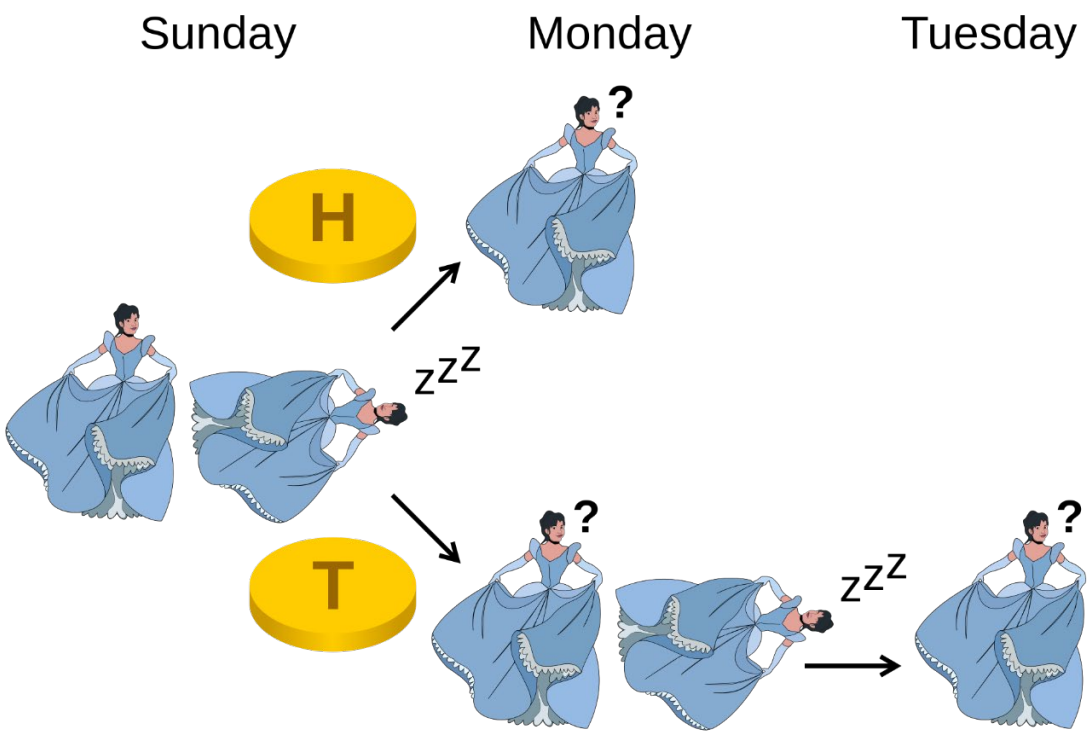

Image: Wikipedia.org; Sleeping Beauty Problem

Problem:

Some researchers are going to put you to sleep. During the two days that your sleep will last, they will briefly wake you up either once or twice, depending on the toss of a fair coin (Heads: once; Tails: twice). After each waking, they will put you to back to sleep with a drug that makes you forget that waking. When you are first awakened, to what degree ought you believe that the outcome of the coin toss is Heads?

Sleeping Beauty: Insurance Example

Charming Insurance Company is analyzing conversion experience on their recent quoting data.

Channel	Age	Quotes	Bound	Conversion Per Quote
Source 1	Youth	2,750	250	9.1%
Source 1	Adult	5,000	500	10.0%
Source 2	Youth	1,500	100	6.7%
Source 2	Adult	1,250	100	8.0%

Channel		Quotes	Bound	Conversion Per Quote
Source 1		7,750	750	9.7%
Source 2		2,750	200	7.3%

	Age	Quotes	Bound	Conversion Per Quote
	Youth	4,250	350	8.2%
	Adult	6,250	600	9.6%

OBSERVATIONS

- Conversion rate is better for Source 1
- Conversion rate is better for adults
  - Observed within each source
- Source 2 attracts relatively more youth

Sleeping Beauty: Insurance Example

- Conversion ratio = $\frac{\#\ of\ policies\ issued}{\#\ of\ quotes}$
- Insured 1 gets three quotes
- Should all three quotes be used in conversion ratio?
- Should just one quote be used in conversion ratio?
- Which quote should be used?

Sleeping Beauty: Insurance Example

- The bulk of the conversion differences were due to the multi-quoting phenomenon!
- The only observation that still holds when considered at unique quote level is that Source 2 attracts relatively more youth

Channel	Age	Quotes	Unique Quotes	Bound	Conversion Per Quote	Conversion Per Unique Quote
Source 1	Youth	2,750	2,500	250	9.1%	10.0%
Source 1	Adult	5,000	5,000	500	10.0%	10.0%
Source 2	Youth	1,500	1,000	100	6.7%	10.0%
Source 2	Adult	1,250	1,000	100	8.0%	10.0%

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	Age	Quotes	Unique Quotes	Bound	Conversion Per Quote	Conversion Per Unique Quote
	Youth	4,250	3,500	350	8.2%	10.0%
	Adult	6,250	6,000	600	9.6%	10.0%

Sleeping Beauty Takeaways

- Ensure data aligns with the question being asked
- Understand the difference between experiment versus outcome
- Recognize when outcomes are not independent

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BERKSON'S PARADOX

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Berkson's Paradox

Berkson's Paradox is a particular kind of **selection bias**, caused by **systematically observing some events more than others**.

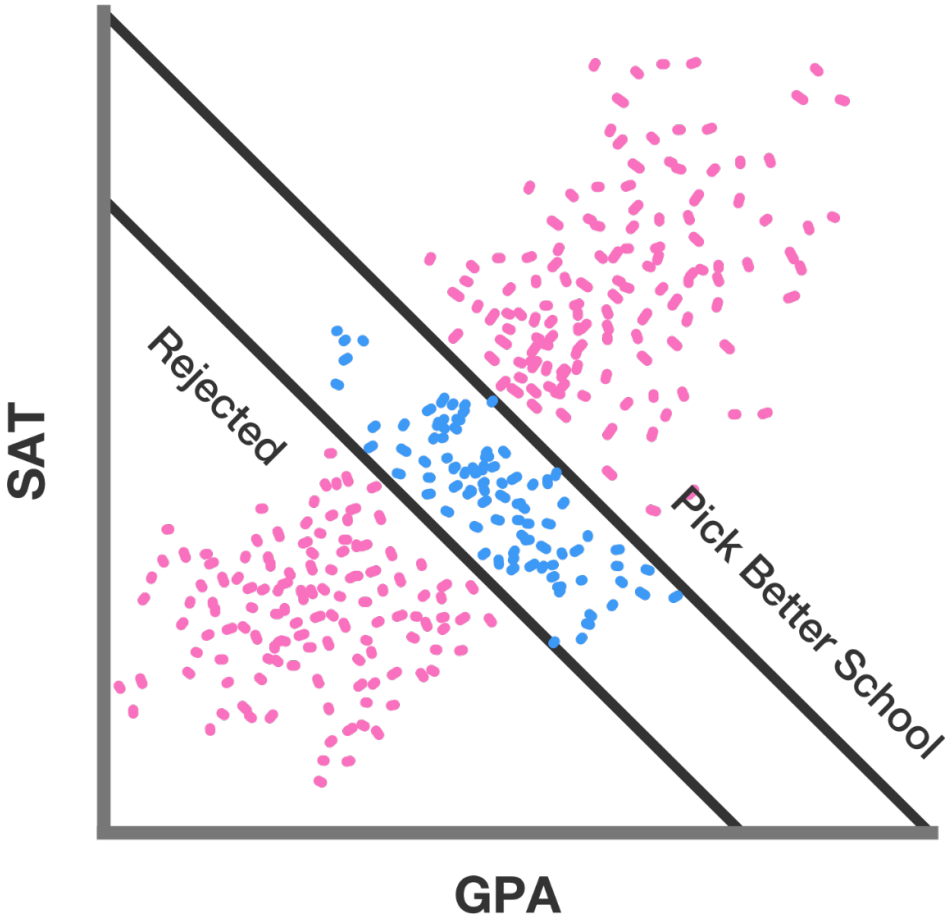
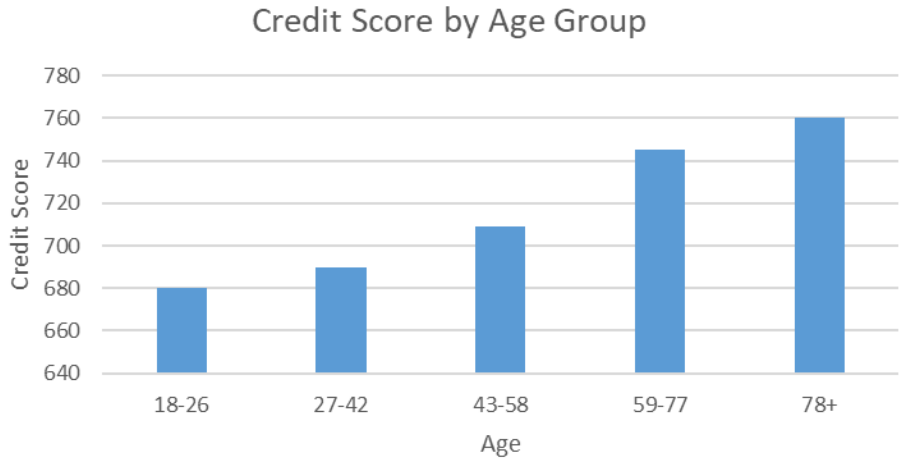


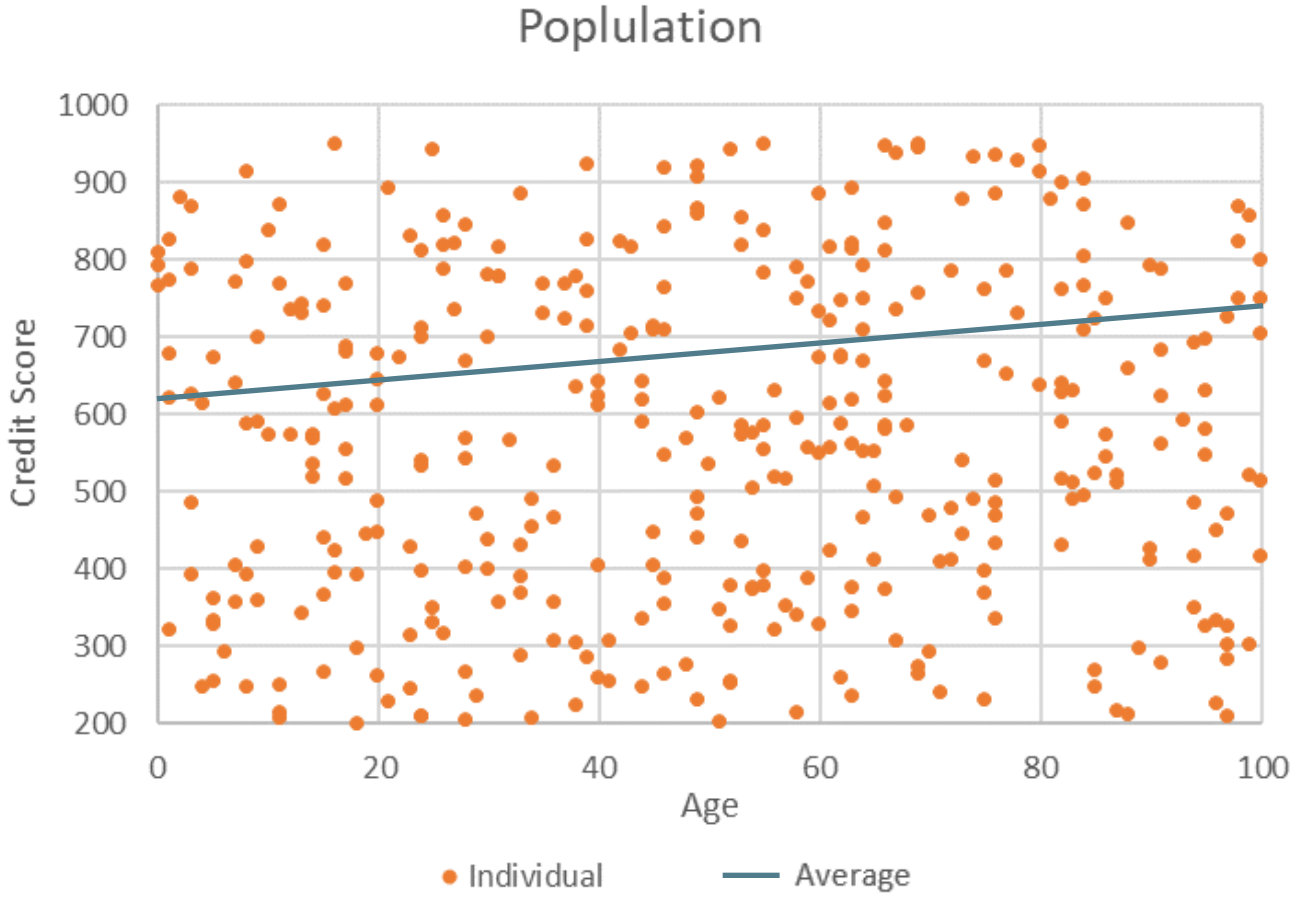
Image: Brilliant Staff Media; Adam Strandberg

Berkson's: Insurance Example

How do age and credit score prevalence in insurance data compare to the overall population?

In the general population, credit score increases as age increases.

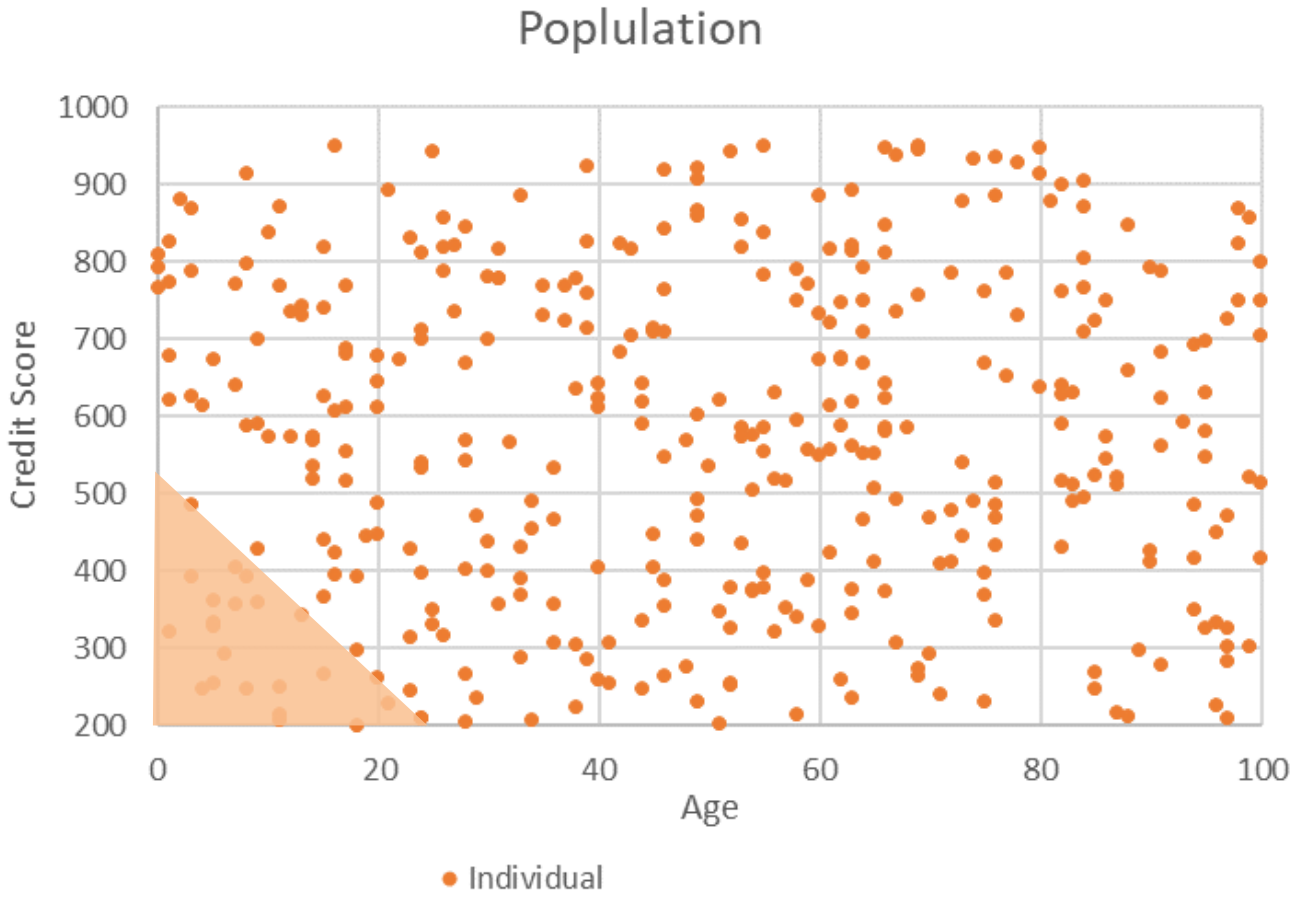




Source: Experian data from Q3 of 2023; ages as of 2023

Berkson's: Insurance Example

- Insurance companies tend to avoid writing young drivers or low credit scores.
- How does this impact the relationship between age and credit score?

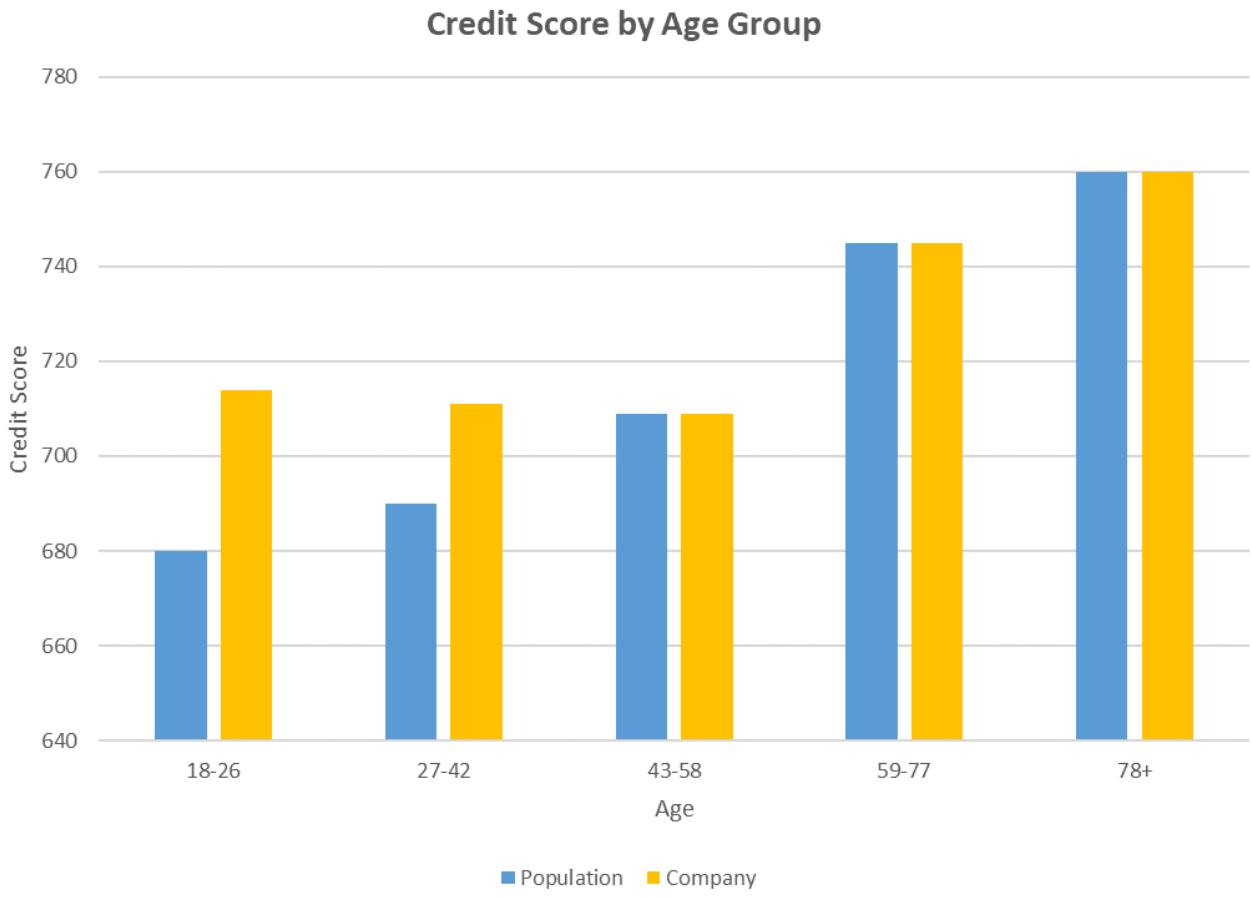


Berkson's: Insurance Example

How do age and credit score prevalence in insurance data compare to the overall population?

POTENTIAL IMPLICATIONS:

- Unintuitive book profile
- Biased model results
- Market blind spots
- Misinformed new business strategy



Berkson's: Takeaways

- Be aware of insurance practices and potential influence on data gathered
- Evaluate whether a sample is representative
- Investigate impacts of sampling bias

Thank You

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