Current Generalized Fractional Floor (GFF)

(1) $rate(s) = max(\kappa + m\ (s - \kappa), s)$

Where:

s is the natively modeled shadow, or unfloored, rate

$\boldsymbol{rate}(\boldsymbol{s})$ is the floored rate as a function of the shadow rate s and the GFF parameters κ and m

$\boldsymbol{\kappa}$ is the threshold parameter – shadow rates below this threshold are subject to the fractional flooring

$\boldsymbol{m}$ is the fraction parameter which applies to the difference $s - \kappa$. Setting *m=0* would imply simple flooring at k, while *m=1* would imply no flooring as *rate(s) = s*

For purposes of GOES, GFF parameters are set to: $\kappa = .004$ and $m = .2$, and the floor applies to the continuous spot rates generated by a 3-factor CIR model

Dynamic GFF – targeting frequency and severity of low rates

The GFF approach can be extended and made more flexible by making the fraction parameter rate dependent and effectively replacing a constant m with $m(s)$ as a function of the unfloored/shadow rate.

(2) $rate(s) = max(\kappa + m(s)\ (s - \kappa), s)$

We assume $m(s)$ can be recast as a piecewise linear function, based on additional targets to explicitly control for:

1. Frequency of negative rates -- threshold at which shadow rate maps to 0 floored rate, $\boldsymbol{s_0}$. This parameter can be set based on x% tail level of the shadow rate distribution, e.g. 1st percentile level based on steady state distribution, to ensure only 1% of the floored rates fall below 0
2. Minimum floored rate boundary, $\boldsymbol{rate_{min}}$ (e.g. -1%) as well as the corresponding shadow rate, $\boldsymbol{s_{min}}$ which can be set at the minimum, or near minimum level observed in the shadow rate distribution.

The dynamic GFF fraction parameter can be defined as follows:

(3) $m(s) = m_0 + max(min(s, \kappa) - s_0, 0)\ R_0 - max(s_0 - max(s,\ s_{min}), 0)\ R_{min}$

where:

$\overline{m}$ is the terminal fraction level that applies when $s = \kappa$

$m_0 = \frac{\kappa}{\kappa - s_0}$ is the fraction that ensures $rate(s_0) = 0$

$$R_0 = \frac{\overline{m} - m_0}{\kappa - s_0}$$

$m_{min} = \frac{\kappa - rate_{min}}{\kappa - s_{min}}$ is the fraction that ensures $rate(s_{min}) = rate_{min}$

$$R_{min} = \frac{m_0 - m_{min}}{s_0 - s_{min}}$$

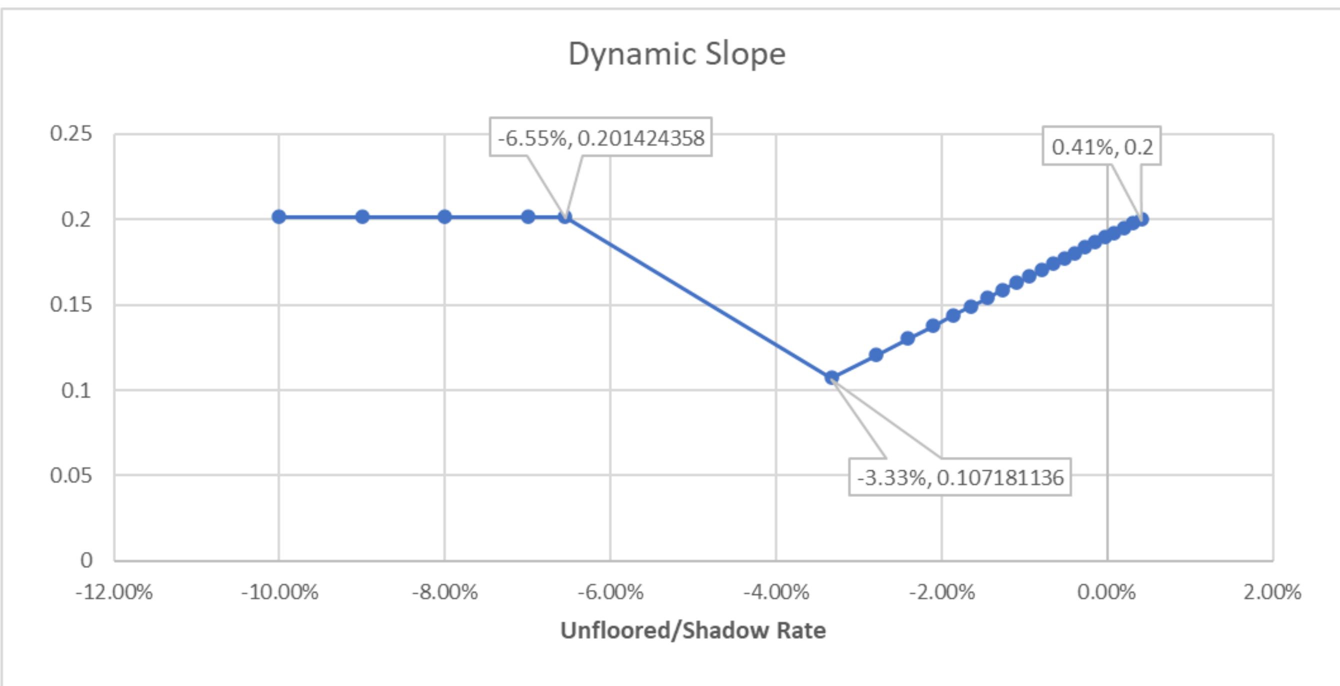
Note that the resulting mapping from shadow rate to floored rate must be an increasing function, so that the terminal fraction parameter is subject to the constraint:

$$\bar{m} < \frac{2\kappa}{\kappa - s_0}$$

Recipe for setting Dynamic GFF parameters:

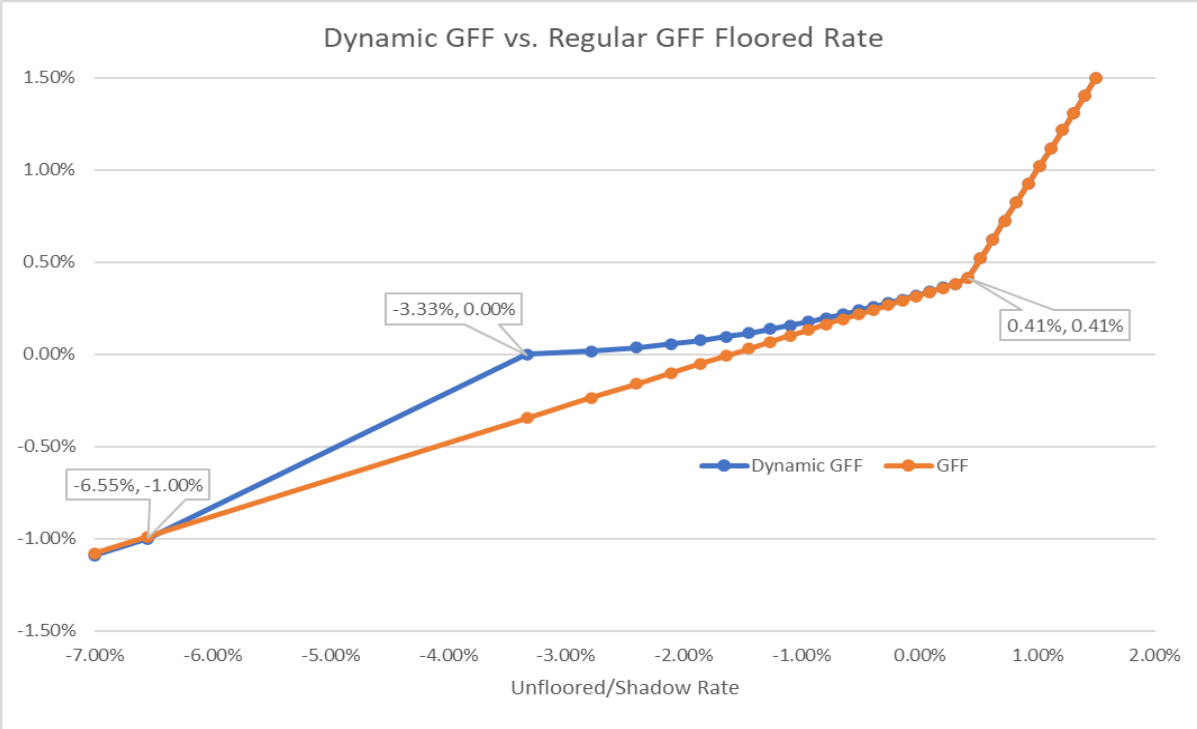
1. Start with the core GFF parameters, $\kappa = .004$ and $\bar{m} = .2$
2. Produce the target distribution of shadow rates as basis for targeting: include tail percentile levels such as minimum, 1%, 2%, etc. and pick the desired short rate tenor, such as 1yr.
3. Target negative frequency: s_0 = -3.3% which is the 1st percentile of the 1yr shadow rate distribution in years 81-100 (steady state) FT2 baseline scenarios. Note that this could also be set to 1.5% or 2% tail levels, to allow for more negative rates in the distribution.
4. Check to see if $\bar{m}$ satisfies the constraint (which it does), and lower accordingly.
5. Set the low rate boundary (the ultimate floor!): $s_{min} = -6.55\%$ (minimum shadow rate in FT2 scenarios) and $rate_{min} = -1\%$. Any other suitable level, like -1.5% would also work. Note that the FT2 baseline scenario 1yr spot rate bottoms out at ~ -1% as well.

This parameterization results in the following dynamic fraction $m(s)$:



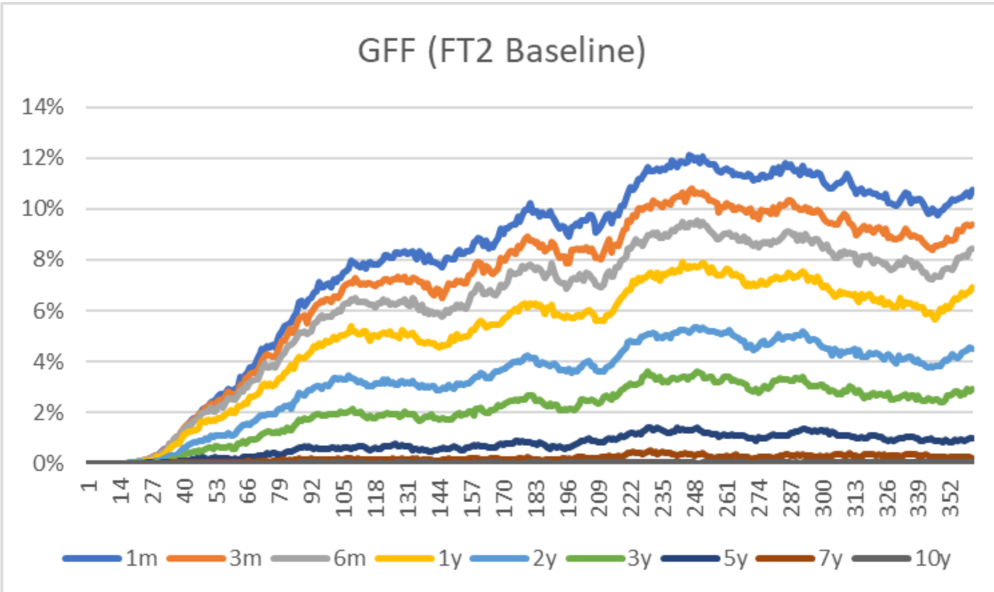
Note the fraction $m(s)$ linearly grades from $\bar{m}$ to m_0 at s=-.0333, to m_{min} at s = .0655 as intended.

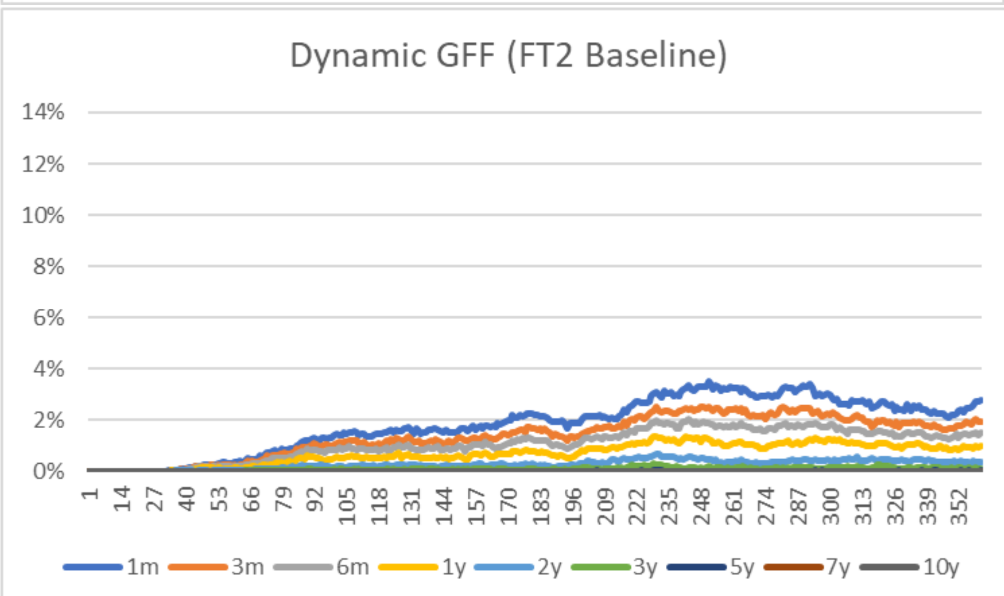
The resulting Dynamic GFF rate mapping is shown below, and compared to the original GFF:



The Dynamic GFF floor hits all the intended targets and produces a reasonable increasing function of shadow rates.

Applying the Dynamic GFF to FT2 Baseline scenarios effectively reduces the frequency of negative short rates (continuous spot rates across 10,000 scenarios over the first 360 months shown). As intended, 1yr spot rate tends to be negative in 1% of scenarios.





Dynamic GFF effectively controls the low tail distribution of short rates, while allowing for a targeted boundary on 1yr rate and a reasonable range of outcomes for other tenors:

	Steady State (80-100yrs) Tail: GFF								
	1m	3m	6m	1y	2y	3y	5y	7y	10y
min	-1.3%	-1.2%	-1.1%	-1.0%	-0.8%	-0.7%	-0.5%	-0.4%	-0.2%
0.5%	-0.6%	-0.6%	-0.5%	-0.4%	-0.3%	-0.2%	-0.1%	0.1%	0.2%
1%	-0.5%	-0.5%	-0.4%	-0.3%	-0.2%	-0.1%	0.0%	0.1%	0.3%
2%	-0.4%	-0.3%	-0.3%	-0.2%	-0.1%	0.0%	0.1%	0.2%	0.3%
3%	-0.3%	-0.3%	-0.2%	-0.2%	-0.1%	0.0%	0.2%	0.3%	0.4%
4%	-0.3%	-0.2%	-0.2%	-0.1%	0.0%	0.1%	0.2%	0.3%	0.6%
5%	-0.2%	-0.2%	-0.1%	-0.1%	0.0%	0.1%	0.3%	0.4%	0.7%
6%	-0.2%	-0.1%	-0.1%	0.0%	0.1%	0.2%	0.3%	0.4%	0.9%
7%	-0.1%	-0.1%	0.0%	0.0%	0.1%	0.2%	0.3%	0.5%	1.1%
8%	-0.1%	0.0%	0.0%	0.1%	0.2%	0.2%	0.4%	0.7%	1.2%
9%	0.0%	0.0%	0.0%	0.1%	0.2%	0.3%	0.4%	0.8%	1.3%
10%	0.0%	0.0%	0.1%	0.1%	0.2%	0.3%	0.5%	0.9%	1.4%

	Steady State (80-100yrs) Tail: Dynamic GFF								
	1m	3m	6m	1y	2y	3y	5y	7y	10y
min	-1.3%	-1.2%	-1.1%	-1.0%	-0.7%	-0.5%	-0.2%	0.0%	0.0%
0.5%	-0.3%	-0.3%	-0.2%	-0.1%	0.0%	0.0%	0.1%	0.1%	0.2%
1%	-0.2%	-0.1%	-0.1%	0.0%	0.0%	0.0%	0.1%	0.2%	0.3%
2%	-0.1%	0.0%	0.0%	0.0%	0.0%	0.1%	0.2%	0.2%	0.3%
3%	0.0%	0.0%	0.0%	0.0%	0.1%	0.1%	0.2%	0.3%	0.4%
4%	0.0%	0.0%	0.0%	0.1%	0.1%	0.1%	0.2%	0.3%	0.6%
5%	0.0%	0.0%	0.1%	0.1%	0.1%	0.2%	0.3%	0.4%	0.7%
6%	0.0%	0.1%	0.1%	0.1%	0.1%	0.2%	0.3%	0.4%	0.9%
7%	0.1%	0.1%	0.1%	0.1%	0.2%	0.2%	0.3%	0.5%	1.1%
8%	0.1%	0.1%	0.1%	0.1%	0.2%	0.3%	0.4%	0.7%	1.2%
9%	0.1%	0.1%	0.1%	0.2%	0.2%	0.3%	0.4%	0.8%	1.3%
10%	0.1%	0.1%	0.1%	0.2%	0.2%	0.3%	0.5%	0.9%	1.4%