SOA Research Institute

Health Care Cost Trends

Graduation of Group Life Waiver of Premium Disability Experience Rates

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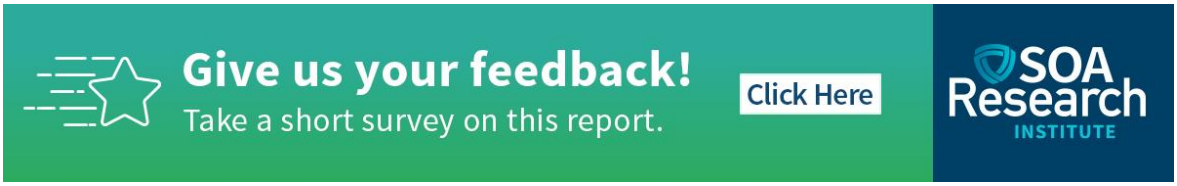
Author R. Jerome Holman FSA, MAAA

Reviewers Matthew Piccolo ASA
Susan R. Sames FSA, MAAA



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Graduation of Group Life Waiver of Premium Disability Experience Rates

Executive Summary

The Society of Actuaries Research Institute (SOA) and the American Academy of Actuaries Group Life Waiver of Premium Valuation Table (GLWPVT) working group was asked by the NAIC Life Actuarial Task force to develop a set of proposed Group Life Waiver of Premium valuation tables for recovery and mortality. The GLWPVT used data collected for a group life waiver of premium experience study.

The goal of the work covered in this paper was to produce graduated experience rates based on the group's predefined set of predictors or input parameters that would correspond to an experience or valuation table structure and the available experience data. The rates produced in this work can be regarded as a foundation for the final experience table but are not complete in that regard and may be different in instances where they have been adjusted by the GLWPVT.

The data for this project was generated from the experience data call of group life waiver of premium recovery and mortality experience that was contributed by twenty companies. The data call was managed by the SOA with the data gathered by a third-party firm. While the experience data included 1999-2016, the work group decided to use the experience from 2006-2016 to build the experience table. The period was limited to those years to capture what was expected to be more relevant experience.

The data was split to develop the experience rates into a training and test dataset. The training data is 70% of all data and the test data is 30%. Only the training data was used to fit models until the final stages of testing model candidates. The range of ages at disability and attained ages was limited to control for credibility and to avoid too high a representation from one or more companies. For the select period analysis that extends to 10 years since disability, the age at disability range was limited to 25-64. Ultimate period rates that are based on attained ages were limited to ages 42-70. Other rates needed for the planned experience or valuation tables would be determined by the working group.

The array of models produced aligned with the table structure defined by the work group. Graduated rates were required for select and ultimate periods for each of recovery and mortality decrements as base rates and with modifiers for diagnosis type. A generalized linear methods (GLM) regression model was used to produce the base rates with open-source R software. The selection of candidate models followed an exhaustive process of testing and re-testing different combinations of various combinations of predictors that were subjected to cross validation analysis. Given the combinations of recovery and mortality, sex and quarterly and annual select periods, eight different models were required for the select period base rates. Four models were required for the ultimate period base rates.

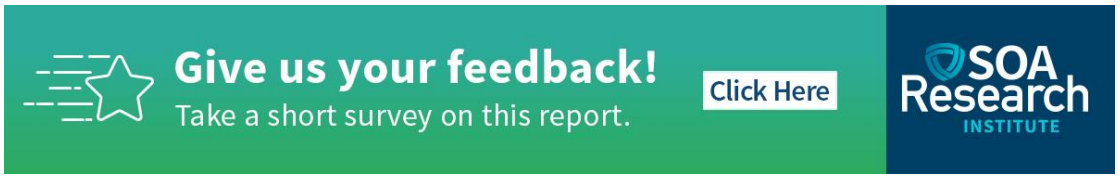
Diagnosis adjustments were developed outside of the GLM modeling process in Excel using a method that based three grades of diagnosis severity group adjustments relative to their aggregate experience. The adjustments were developed without regard to sex for the select and ultimate periods. They vary by duration during the select period and by attained age during the ultimate period. Diagnosis types with insufficient actual diagnosis information (the No Diagnosis group) were deemed a fourth diagnosis group that used the base rates without adjustment.

The base rate models produced excellent fits to the data. The base rate model regression adjusted R squared values ranged from .904 to .987. All mortality models had lower scores than recovery due to the pattern of the underlying bumpier experience rate pattern than for recoveries. When viewed from an actual to expected perspective, all base rate models, select and ultimate, had virtually zero average residual. The average exposure weighted graduated and actual rates were virtually equal for all base rate models.

The residuals for the base rates modified by diagnosis adjustments varied across the diagnosis groups. But when all the diagnosis groups' residuals based on their own diagnosis adjusted fit rates and their actual rate experience were combined, the aggregate result was very low residuals as per the base rates for both recovery and mortality and both select and ultimate periods.

The No Diagnosis group had positive residuals because it had lower recovery and slightly lower mortality than the aggregate of all experience. The converse is also true. Because the other the other three diagnosis severity groups combined had higher recovery and mortality (except select period mortality) than the aggregate of all experience corresponding to the base rates and their adjustments were centered on their combined experience, their residuals were generally negative. This means that their recovery and mortality rates were both understated but because those are offsetting risks and the effect was stronger on recoveries than mortality, the working group decided the use of their rates together was reasonable.

The potential variability of the exposure weighted graduated base rates from their true statistical population values was estimated with a bootstrapping technique. These variations appeared to be reasonably well confined to produce reliance on the base rates with small margins to allow for the variations. Expressed as a percentage of the actual average rate, the maximum 95% plus/minus confidence interval range was 1.214% and 1.648% for select period recoveries and mortality, respectively and 3.809% and 2.772% for ultimate period recovery and mortality, respectively.



Section 1: Background

The Society of Actuaries Research Institute (SOA) and the American Academy of Actuaries Group Life Waiver of Premium Valuation Table (GLWPVT) working group was asked by the NAIC Life Actuarial Task force to develop a set of proposed Group Life Waiver of Premium valuation tables for recovery and mortality. The GLWPVT used data collected for a group life waiver of premium experience study. That data was used to initially assess potential predictors that could be used as inputs in an experience table.

Because the valuation table based on the prior generation of this experience table only used sex and age at disability for select period rates and sex and attained age for ultimate rates the GLWPVT decided to limit the number of parameters in transitioning to a table with more inputs than previously used. That decision was to add grouped diagnosis types as an additional delineation of table rates. On that basis, the GLWPVT produced some preliminary rates with work that was done completely in Excel. Given the limitations of Excel to easily produce analysis using generalized linear methods (GLM) of regression the GLWPVT built on and extended this analysis to produce rates with a more transparent and documented process. For that purpose, R was used as the main tool to produce and analyze GLM regression fits of the data.

Essentially, the goal of this work was to produce graduated experience rates based on the group's predefined set of predictors or input parameters and the available experience data. Although they were tested extensively for consistency, the working group retained the option to adjust the rates. Additionally, the GLWPVT planned to extend the table rates for those ages which were not covered by the scope of this work. The rates produced in this work can be regarded as a foundation for the final experience table but are not complete in that regard and may be different in instances where they have been adjusted by the GLWPVT.

Section 2: Data Profile

The data for this project was generated from the experience data call of group life waiver of premium recovery and mortality experience that was contributed by twenty companies. The data call was managed by the SOA with the data gathered by a third-party firm. While the experience data included 1999-2016, the work group decided to use the experience from 2006-2016 to build the experience table. The period was limited to those years to capture what was expected to be more relevant experience.

The graphs below in Figures 1-4 show a summary of the exposure by count and decrement counts in the experience data for 2006-2016. Because exposure is nearly identical for recovery and mortality experience, recovery exposure is used here to represent the exposure for both decrements. The total exposure for ages less than twenty-five and greater than or equal to sixty-five is less than 1% of total exposure. Because of concerns about credibility and over reliance on any one contributing company's data, those ages were excluded by the work group from the analysis of fitting rates and are excluded in the graphs below. Experience by duration since disability was gathered on a quarterly basis in the first two years and annually thereafter. The graphs below with respect to duration since disability annualize the quarterly exposure and counts for consistency with the later years shown.

Figure 1

EXPOSURE COUNT BY DISABILITY AGE GROUP AND SELECT VS. ULTIMATE PERIODS

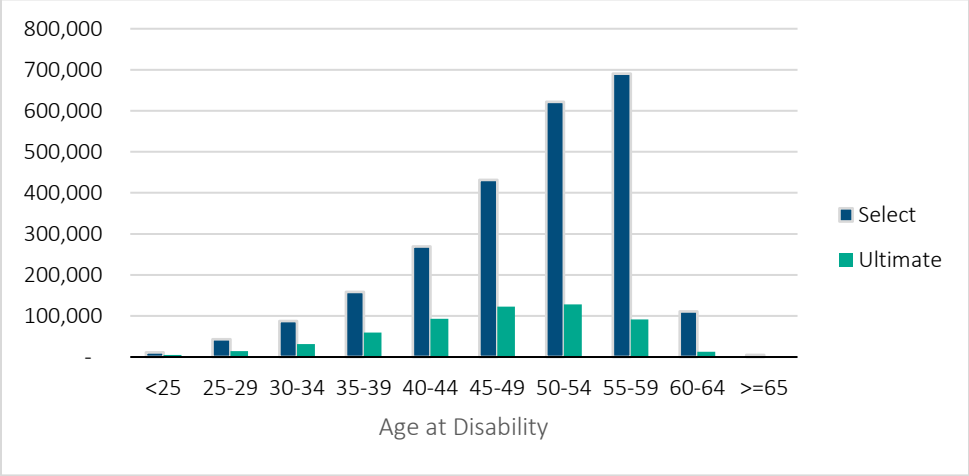


Figure 2

EXPOSURE COUNT BY DURATION

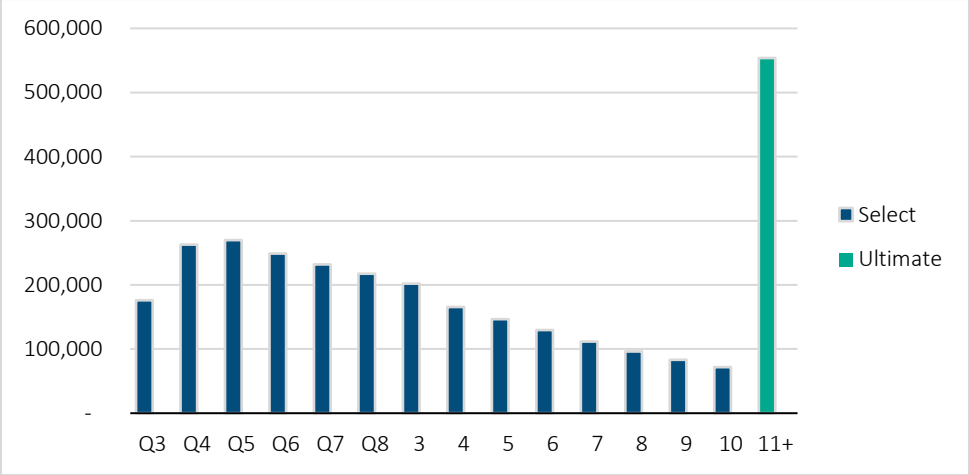


Figure 3
RECOVERY AND DEATH COUNTS BY DISABILITY AGE GROUP AND SELECT VS. ULTIMATE PERIODS

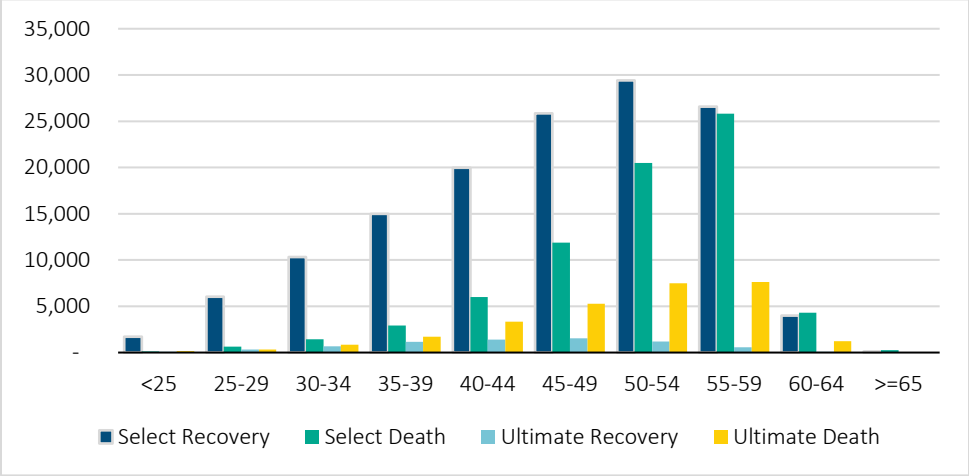
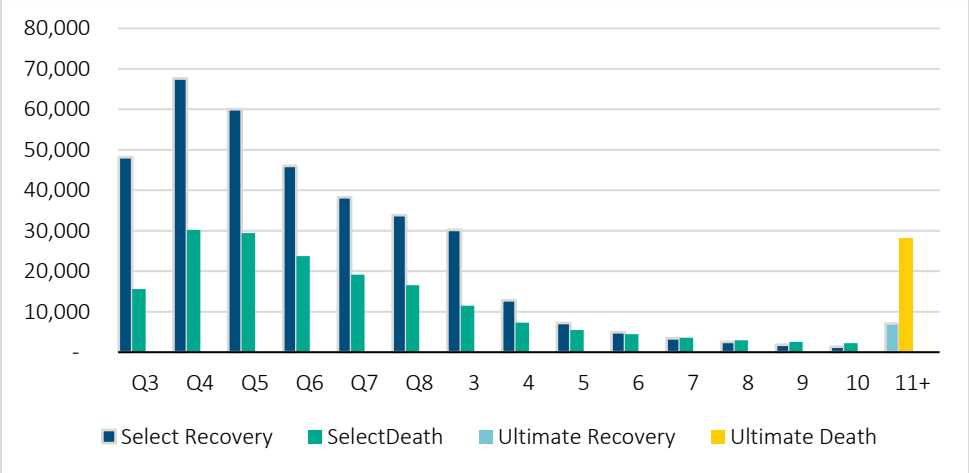


Figure 4
RECOVERY AND DEATH COUNTS BY DURATION



Section 3: Modeling

3.1 DATA SELECTION

The data was split to develop the experience rates into a training and test dataset. The training data is 70% of all data and the test data is 30%. Only the training data was used to fit models until the final stages of testing model candidates. The allocation of the data to the training and test subsets was done randomly with two constraints to preserve the character of it. First, the random assignments to training or test data were done within cells of disability age group and sex combinations. The second constraint was to preserve the case character of any participant's disability. This was done by identifying participant case records and assigning them in total to either the training or test data.

The range of ages at disability and attained ages was limited to control for credibility and to avoid too high a representation from one or more companies. For the select period analysis that extends to 10 years since

disability, the age at disability range was limited to 25-64. Ultimate period rates that are based on attained ages were limited to ages 42-70.

Grouped ages or alternatively individual disability ages were considered for fitting the select period rates. Individual ages offered the potential of smoother patterns whereas grouped ages were considered by the work group to have a greater likelihood of more credible results. The work group agreed grouped ages would be a better approach after seeing a lower exposure weighted residual error measurement on the grouped ages. That decision was reinforced by the more logical flow of the rates being determined in line with valuation table five-year age groups. Potential concerns about a variance of the central age within age groups across companies was addressed by reviewing the exposure weighted average age at disability for each group within each company. Table 1 shows the minimum, mean, maximum and standard deviation of the companies' age group average ages. These values show relatively small differences across the companies.

Table 1
AGE DISTRIBUTION BY AGE GROUPS ACROSS CONTRIBUTING COMPANIES

Age Group	24-29	30-34	35-39	40-44	45-49	50-54	55-59	60-64
Minimum Mean	26.28	32.09	37.15	41.00	47.08	52.00	56.67	60.08
Mean	27.42	32.32	37.26	42.21	47.17	52.10	56.87	60.55
Maximum Mean	28.31	32.61	37.68	42.34	47.68	52.79	57.17	60.94
Std Dev of Means	0.35	0.14	0.12	0.27	0.12	0.16	0.12	0.27

3.2 MODEL SELECTION

The array of models produced aligned with the table structure defined by the work group. Graduated rates were required for select and ultimate periods for each of recovery and mortality decrements as base rates and with modifiers for diagnosis type. The select period encompasses quarters 3-8 and years 3-10 since the occurrence of disability. Base rates were determined by sex and duration since disability for select periods and by attained age for ultimate experience. Diagnosis table adjustments to the base rates by diagnosis group were based only on duration since disability in the select period and by five-year attained age groups in the ultimate period.

A generalized linear methods (GLM) regression model was used to produce the base rates. Because causes of differentiated recovery and mortality experience were thought to be known and outputs (response variable values) were known preference was given to supervised rather than unsupervised models[1]. Brief consideration was given to a logistic model but when tested in parallel with GLM results it did not perform as well. A generalized additive model (GAM) could have been another choice but a nuance of quarterly vs. annual rates in the select period made modeling across all select periods at once difficult. The main advantage of a GAM model would have been to capture the irregular pattern of recoveries crossing from quarter eight to year three. But that was negated by the necessity of splitting the analysis due to the different basis of exposure in the quarterly vs. annual periods. Using a GLM approach split across two periods gave the flexibility of a GAM regression at the split across the quarterly and annual periods while also providing more insight on the relative contributions of the predictors as defined by their regression coefficients that are not directly observable under GAM approaches.

A GLM Poisson model with log link was used to produce the rates. A primary assumption of this method is that the decrement counts produce equal conditional (e.g., with respect to a specific cell like disability age and duration) means and variances. To validate this assumption, the mean and variance of the training data decrement counts, recovery and mortality, were reviewed. Given the five-year age groups and select durations for both sexes combined the mean and variance of the counts were determined for each cell for both decrements. The ratio of the variance to the mean averaged across all cells was 1.037 for recovery and 1.057 for mortality. The near average equality of the mean and variance satisfies a primary Poisson

regression assumption[2]. Even though most of the conditional distributions did not satisfy a dispersion test to gauge Poisson distribution likelihood, the regressions performed well in fits of the graduated to actual rates.

In addition to the quarters 3-8 and years 3-10 duration split of the models, the number of models produced for the base rates was also affected by a decision to fit the experience of each sex based only on its own experience. This is a subtle but important difference in modeling the rates. Although male and female experience can be modeled together with resulting differentiated rates between them, each will produce different rates than if they are modeled separately capturing their experience alone. Because the work group wanted to capture the true experience in the modeled rates for both sexes they were modeled separately.

The decisions on modeling for sex and duration for the select period produced four models for each of recovery and mortality. Because the base rates table inputs or structure was limited to age at disability and duration since disability model fitting was limited to those predictors or variants of them. A large variety of age and duration related predictors were tested. These involved powers of age and duration, log of duration and modifiers to age or duration, e.g., a modifier such as age less than 30. The set of predictors tested also included the effect of introducing experience rates from the 2005 Group Life Waiver of Premium table. Whether these should be included was discussed extensively by the work group. The advantage of using them is that a similar pattern to current experience could help improve the fit. The concern about using them is that they could inappropriately influence the fit rates relative to their actual experience. It does not appear that an inappropriate influence occurred because the GLW 2005 rates were used as predictors in only two of the eight models (recovery rates years 3-10 for males and females).

The selection of models as candidates followed an exhaustive process of testing and re-testing different combinations of the predictors described above. Given the combinations of recovery and mortality, sex and quarterly and annual select periods, eight different models were required for the select period base rates. Acceptance or rejection criteria were based on the Akaike Information Criterion (AIC), mean squared error and the square root of the mean of the exposure weighted residuals. Generally, but not always, these measures had the same ranking when viewed across model candidates. Where rank differences occurred, greater weight was given to the exposure weighted residuals because that measure was thought to relate directly to the intended use of the rates. It was derived from the residuals of the fit and actual rates from each of the disability age and duration cells. Thus, much as a valuation actuary would likely be concerned about the total reserve rather than the reserve of an individual participant, this measure produces an exposure weighted residual value that parallels the interest of the valuation actuary to have a good exposure weighted fit of the predicted rates. Acceptance of a model candidate was also conditioned on statistical significance of the predictors. In some instances, a set of predictors produced an otherwise attractive fit, but one or more predictors did not have a significant p value of less than .05. Because those combinations could have been overfitting with a flawed predictor, they were rejected.

Each of the eight sets of model candidates that passed the screens described above, which were each as many as four to five stages, were also subjected at the same time to a two-stage validation process. The validation process used the measures described above as the main inputs for judgment. The first step was a cross validation using the training data. In that process the training data was subdivided into 5 or 10 randomly chosen segments. As applicable 5 or 10 fits of rates were conducted by taking the 4 or 9 segments not chosen as a “training” data set and then testing the results against the remaining segment. The statistical measures, e.g., AIC and the weighted residual were then averaged across the 5 or 10 fits. Then that process was repeated for a total of 10 rounds and each round was averaged to obtain a final average measure to score each model candidate. The models that appeared as the leaders of cross validation were promoted to the final testing stage with the test data. This is data that the models had not previously seen and was intended to weed out instances of overfitting that might have occurred in the prior stages of model selection. The models with the best scores were deemed as the final models.

A similar process was followed for initial candidate testing, to cross validation and final testing for the ultimate base rates. But because the predictors were limited to variations of attained age as predictors, the process was much more abbreviated relative to the select rates.

The list of predictors that were used in one or more of the candidate models for the select and ultimate base rates are carried in the R code of the base rates script (see Section 8: R Code for script files).

Section 4: Base Rates

The selected model from the test validation was run to re-fit the rates to the merged training and test data (the full data set) to produce the final rates. A comparison of those rates to the actual rates are shown graphically in Appendix A and are listed in Appendix B. Table 2 below shows a summary of the degree of the model fits for each of the eight base rate select period models. The adjusted R^2 values shown were derived directly from the unweighted residuals of the fit vs. actual rates. The weighted residuals are the square root of the exposure weighted residuals discussed in Section 3 Modeling which were used to guide the selection of candidate models. The adjusted R^2 values are very high indicating a good fit of the graduated rates to the actual rates. The mortality values are less than the recovery ones which is a byproduct of the choppier nature of the mortality rates.

Table 2
AGE DISTRIBUTION BY AGE GROUPS ACROSS CONTRIBUTING COMPANIES

Decrement	Sex	Duration	Adjusted R^2	Weighted Residual	Average Actual Rate	Residual as % of Actual Rate
Recovery	Female	Quarters 3-8	0.982	0.3280%	5.73%	5.7%
Recovery	Female	Years 3-10	0.987	0.4441%	6.94%	6.4%
Recovery	Male	Quarters 3-8	0.961	0.3064%	4.66%	6.6%
Recovery	Male	Years 3-10	0.981	0.3840%	5.66%	6.8%
Mortality	Female	Quarters 3-8	0.959	0.1358%	2.20%	6.2%
Mortality	Female	Years 3-10	0.938	0.2396%	3.57%	6.7%
Mortality	Male	Quarters 3-8	0.934	0.1723%	2.67%	6.4%
Mortality	Male	Years 3-10	0.904	0.2604%	4.43%	5.9%

The base rates were reviewed extensively for consistency by age, duration since disability and sex. Anomalies noted relative to normal expectations of the progression of recovery and mortality rates for individuals with disabilities were investigated. That analysis involved cross checks with the actual data, reference to the Group Long Term Disability 2008 Experience Table (GLTD 2008) report and consideration of the limitations of merging the separately derived quarters 3-8, years 3-10 and ultimate rates. Where rate progressions were supported by the data or similar patterns in the GLTD 2008 table the work group expected to retain the rates as produced. Observations relative to duration consistency and female to male rate consistency are discussed below.

4.1 DURATION

4.1.1 RECOVERY RATES QUARTER 8 VS. YEAR 3

The fit or regressed recovery rates were higher for some age ranges for both males and female in year 3 than quarter 8. Normally, recovery rates would be expected to decrease with duration. This was a particular area of focus because the noted progression occurred at the merge point of the separately

derived regression models. Because the fit rates aligned with the progression of the data and were consistent with the GLTD 2008 table, the fit rates in this region were retained as produced. The work group discussed the likelihood that the change of definition of disability from own occupation to any occupation could be the main cause of this. But because of varying mixes of types of policies across the contributing companies regarding that definition it was not specifically validated.

4.1.2 SELECT AND ULTIMATE TRANSITION

The transition or bridge between the select and ultimate recovery rates was noted to be too abrupt a decrease for the lower ages. The work group discussed this and planned to adjust those rates in its review process. There were no similar concerns for the bridge of the mortality rates.

4.1.3 MORTALITY QUARTER 4

While the general mortality pattern is for rates to decrease during the select period, quarter 4 shows a sharp increase from quarter 3. This corroborates with the data. The work group discussed this and considered it reasonable considering likely delayed reporting of mortality within the first year after disability.

4.2 FEMALE VS. MALE RATES

4.2.1 SELECT PERIOD

Generally female recovery rates were higher than male rates while the opposite occurred for mortality. That aligned with expectations. But because the tables by sex were generated separately there was a risk of inconsistency between them. Those areas that did not follow the normal expectation were reviewed. Regarding recovery the general relationship does not hold for all diagnosis types. The GLTD 2008 study showed that females had lower recovery rates for circulatory, digestive, musculoskeletal and injury other than back diagnoses. Because the fit rates showed relatively low occurrence of lower female than male rates, that were not much below the male rates and almost entirely concentrated in durations 8-10, they were not a concern. Female mortality rates were lower than male rates in all but one select period age-duration cell (age group 25-29: duration 4). The group discussed the possibility of smoothing that value which likely was the byproduct of limited exposure leading to volatile nearby mortality that influenced the fit.

4.2.2 ULTIMATE PERIOD

Ultimate rates were modeled for attained ages 42-70. For those ages the recovery rates showed a pattern opposite of the select period where male rates were higher at the lower end of the range until about the mid-fifties where rates were roughly the same. This result was checked against the data which had a similar pattern. Although it cannot be construed as a direct correspondence, the GLTD 2008 recovery rates also show an increasing tendency for male rates to exceed female ones with increasing duration past 10 years. The male mortality rates were higher than the female rates for all ages which was the expected result.

Section 5: Diagnosis Adjustments

The effect of the disability diagnosis on the likelihood of recovery and mortality was the only other indicator than sex, age and duration chosen by the work group to determine a recovery or mortality rate. To aid in producing credible data and for practical implementation reasons the seventeen submitted diagnosis types in the experience data were consolidated into four groups for each of recovery and mortality rate adjustments.

The diagnosis types submitted by contributors to the study and their translation to the four diagnosis types are shown in Table 3 below.

Table 3
DIAGNOSIS TYPE GROUPS

Diagnosis Type	Recovery	Mortality
Alcohol & Drug	High	Low Non-Cancer
Back	Medium	Low Non-Cancer
Cancer	Medium	Cancer
Circulatory	Low	Low Non-Cancer
Diabetes	Low	High Non-Cancer
Diagnosis not provided	No Diagnosis	No Diagnosis
Digestive	Medium	High Non-Cancer
Ill-defined and Misc. Conditions	Low	Low Non-Cancer
Infectious Diseases	Medium	Low Non-Cancer
Injury other than back	High	Low Non-Cancer
Invalid	No Diagnosis	No Diagnosis
Maternity	High	Low Non-Cancer
Mental & Nervous	High	Low Non-Cancer
Nervous System	Low	Low Non-Cancer
Other	Low	High Non-Cancer
Other Musculoskeletal	High	Low Non-Cancer
Respiratory	Low	High Non-Cancer

It should be noted that the diagnosis types of Diagnosis not provided and Invalid (as in not a validly submitted code) that were combined and labeled as the "No Diagnosis" group is considered a diagnosis group even though it uses the base rates without modification. The work group decided to use the base rates without modification for the No Diagnosis group for simplicity of the method and acknowledged that even though the No Diagnosis group had lower recovery and slightly lower mortality than the base rates those offsetting risks permitted the use of the base rates without modification for them.

The diagnosis adjustments were analyzed for differences when developed without regard to sex vs. a sex distinct basis. One set of factor adjustments for both sexes had the appeal of a simpler structure whereas sex distinct adjustment factors would align with having captured differences by sex in the underlying base rates. Differences of the adjustment factors and their residuals (differences of the diagnosis adjusted fit rates to the actual diagnosis group rates) were reviewed for each basis. After reviewing this information, the work group decided that although there were improved residuals using sex distinct adjustments, the amount of their decrease and the limited material differences of adjustment factors for the two bases did not warrant introducing the complexity of a set of factors for each sex. Accordingly, the adopted diagnosis adjustment factors are unisex in nature but are applied to the sex distinct base rates.
The diagnosis adjustments were developed separately for the select and ultimate periods. They vary by duration during the select period and by attained age during the ultimate period. The adjustments were developed by evaluating the experience of the three gradations of recovery or mortality likelihood relative to their aggregate experience along the dimensions of duration for the select period and attained age for the ultimate period. This approach assumes that the experience of those three groups is the same as the experience underlying the base rates. That experience differs slightly for recovery and to a smaller degree for mortality. The differences are small enough to permit the approach used. The adjustments are shown following the base rates in Appendix B.

Section 6: Validation

Validation was performed for all variations of the graduated or fit rates (select period durations through year 10 and ultimate period attained ages 42-70) vs. the actual rates. The purpose of this validation was to measure the exposure weighted sum of residuals to understand model variance as applied to the experience rather than using validations to select a model. The actual rates and exposure were based on count which is consistent with the development of the graduated rates. Measures of the exposure weighted residual, that residual as a percentage of the average rate and the exposure times the weighted residual were developed. For this purpose, the residual was based on a comparison of the fit minus actual rates. Table 4 below shows a condensed view of these measures for the base rates and diagnosis adjusted rates where all diagnosis groups are combined.

The base rates in Table 4 without modification for diagnosis adjustments measured across all exposure (consistent with the method for developing the graduated base rates) showed an excellent aggregate fit. The weighted residuals are virtually zero for all base rate variations. The residuals for the base rates modified by diagnosis adjustments varied across the diagnosis groups. But when all the diagnosis groups' residuals based on their own diagnosis adjusted fit rates and actual rate experience were combined, the aggregate result was very low residuals as per the base rates for both recovery and mortality and both select and ultimate periods.

Table 4

BASE RATE AND DIAGNOSIS ADJUSTED RESIDUAL MEASURES COMBINED BY SEX AND DIAGNOSIS TYPE

Rate Basis	Decrement	Duration	Weighted Residual	Residual % of Actual Rate	Exposure	Residual*Exposure
Base Rate	Recovery	Select	0.00%	0.00%	1,358,824	(0.0)
Base Rate	Recovery	Ultimate	0.00%	-0.02%	390,107	(1.0)
Base Rate	Mortality	Select	0.00%	0.00%	1,345,098	0.0
Base Rate	Mortality	Ultimate	0.00%	0.00%	392,395	(0.0)
Diagnosis Adjusted	Recovery	Select	0.10%	1.01%	1,358,824	1,380.2
Diagnosis Adjusted	Recovery	Ultimate	0.01%	0.40%	390,107	22.1
Diagnosis Adjusted	Mortality	Select	0.06%	1.10%	1,345,098	811.4
Diagnosis Adjusted	Mortality	Ultimate	0.01%	0.18%	392,395	23.2

Table 5 below shows the variation by diagnosis type of the residuals for both sexes combined. The residuals (weighted residual or residual percentage of the actual rate) showed a pattern that had positive residuals (fit is greater than actual rate) for the No Diagnosis group and generally negative residuals for the other groups based on three gradations of likely recovery or mortality for reported diagnoses. The No Diagnosis group had positive residuals (as discussed in Section 5) because it had lower recovery and slightly lower mortality than the aggregate of all experience. The converse is also true. Because the other the other three diagnosis types combined had higher recovery and mortality (except select period mortality) than the aggregate of all experience and their adjustments were centered on their combined experience, their residuals were generally negative. Appendix C shows expanded detail on these measures by sex. Although they vary more than the measures that are combined by sex shown in Table 5 that variation is to be expected because the diagnosis adjustments were developed on a combined sex basis.

Table 5

DIAGNOSIS TYPE RESIDUAL AS A PERCNTAGE OF ACTUAL RATE RANGE

Decrement	Duration	Diagnosis Type	Weighted Residual	Residual % of Actual Rate	Exposure	Residual*Exposure
Recovery	Select	No Diagnosis	1.52%	19.41%	320,408.1	4,883.7
		Low	-0.54%	-8.40%	456,613.3	(2,459.2)

		Medium	-0.59%	-4.46%	341,181.8	(2,010.3)
		High	0.40%	2.56%	240,620.5	965.9
Recovery	Ultimate	No Diagnosis	0.26%	22.41%	169,600.3	436.0
		Low	-0.15%	-11.98%	91,709.9	(142.1)
		Medium	-0.29%	-13.68%	60,901.5	(177.2)
		High	-0.25%	-13.13%	55,689.5	(136.5)
Mortality	Select	No Diagnosis	0.23%	4.44%	318,652.5	722.2
		Low Non-Cancer	-0.02%	-0.97%	748,529.7	(163.5)
		High Non-Cancer	0.04%	0.66%	166,138.0	59.5
		Cancer	0.17%	0.62%	111,777.8	193.3
Mortality	Ultimate	No Diagnosis	0.36%	12.31%	185,148.0	667.4
		Low Non-Cancer	-0.26%	-9.40%	157,581.6	(405.4)
		High Non-Cancer	-0.44%	-8.62%	29,421.5	(128.6)
		Cancer	-0.54%	-8.03%	20,244.4	(110.2)

Section 7: Rate Variability

Regression is a process which itself is subject to a variable result depending on the data supplied to it. The data supplied by the contributors to the study is a sample of industry data. Even if it were 100% of the industry and no conditions changed, a resampling of that data at a future point would be expected to produce different results due to natural statistical variations. These variations could cause a decreased degree of fit of the graduated to actual rates experienced at this later point in time. Allowing for these possible variations is a criterion that could be used as one criterion in considering rate margins.

This potential variability was explored through re-sampling of the data with the bootstrapping technique. This method treats the experience data gathered as an estimate of the population. Resampling the data with replacement mimics an estimate of the population. When the process is repeated many times the range of results provides an indication of the variability of the estimate of the population value. In this regard "the population is to the sample as the sample is to the bootstrap samples"[3]. Relative to this work, the aim was to re-use data with replacement to gauge the potential variability of modeled actual rates against the fixed graduated rates.

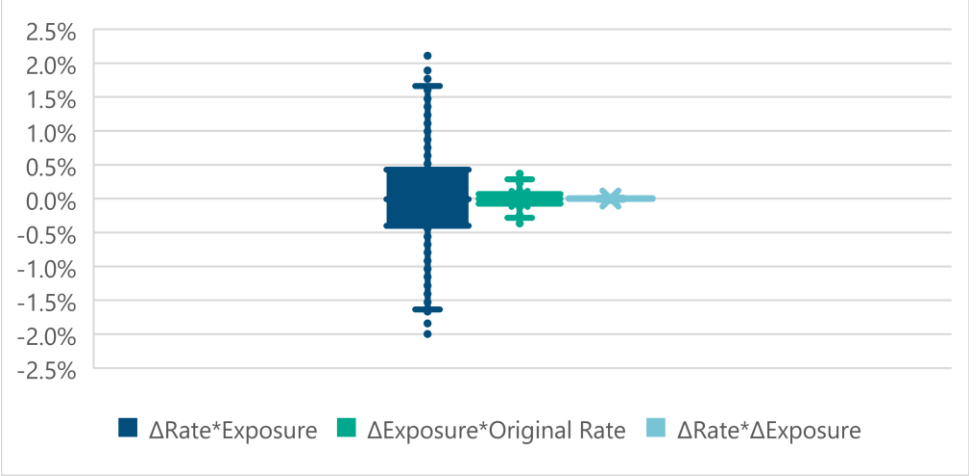
The change in the average rate from one period to the next can be decomposed as the sum of the following:

1. Δ_{Rate}*Exposure
2. $\Delta_{Exposure}$*Original Rate
3. Δ_{Rate}*$\Delta_{Exposure}$

The difference of the original and bootstrap modeled average rates is the exposure weighted sum of the three values above for each cell of age at disability and duration for select periods and by attained age for ultimate periods. Figure 5 shows this distribution for male recoveries durations years 3 -10. Most of the rate differential is due to element (1) above.

Figure 5

ATTTRIBUTION OF RECOVERY MALE DURATIONS 3-10 BOOTSTRAPPED RATE CHANGE



Regarding total average rate differences, element (2) is not a concern. That is because a shift of exposure would not affect the adequacy of an expected rate for a given cell of age or age and duration. Element (3) is comprised of both effects of different exposure and rate. To be conservative it has been added to element (1) in the determination of potential rate variability in this analysis. Figures 6 and 7 below show rate variability as a percentage of the average graduated rates for recovery and mortality, respectively.

Figure 6

BOOTSTRAPPED RECOVERY RATE VARIABLILTY AS A PERCNTAGE OF ACTUAL RATES

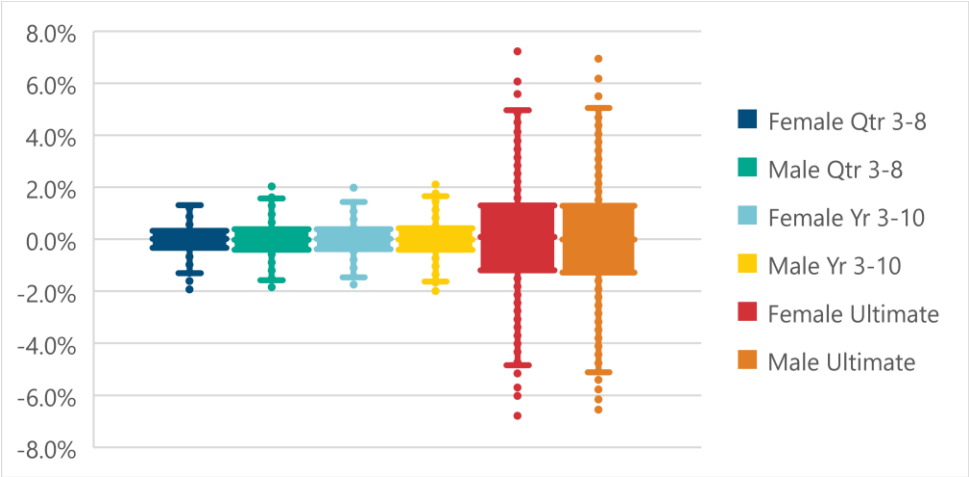


Figure 7
BOOTSTRAPPED MORTALITY RATE VARIABLILTY AS A PERCNTAGE OF ACTUAL RATES

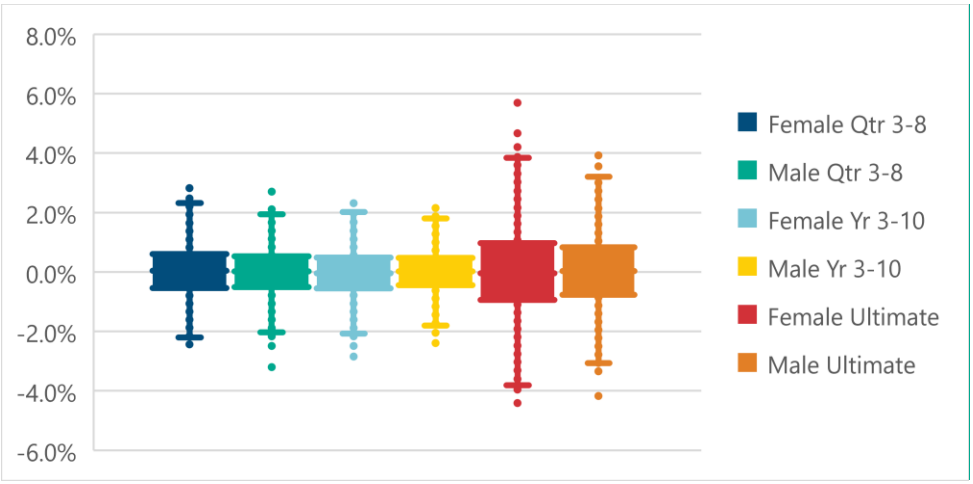


Table 6 below shows the associated 95% confidence intervals of the rate variability. These variations appeared to be reasonably well confined to produce reliance on the base rates with small margins to allow for the variations. Expressed as a percentage of the actual average rate, the maximum 95% confidence interval low/high absolute value was 1.214% and 1.648% for select period recoveries and mortality, respectively and 3.809% and 2.772% for ultimate period recovery and mortality, respectively.

Table 6
POTENTIAL AVERAGE BASE RATE VARIABLITIY AS A PERCENTAGE OF ACTUAL AVERGE RATE

Decrement	Sex	Duration	Bias	Std Dev Std Estimate	95% CI Low	95% CI High
Recovery	Female	Quarters 3-8	0.006%	0.495%	-0.977%	0.965%
Recovery	Male	Years 3-10	-0.001%	0.589%	-1.153%	1.156%
Recovery	Female	Quarters 3-8	0.005%	0.547%	-1.078%	1.068%
Recovery	Male	Years 3-10	0.011%	0.614%	-1.214%	1.193%
Recovery	Female	Ultimate	0.049%	1.885%	-3.745%	3.647%
Recovery	Male	Ultimate	-0.002%	1.941%	-3.805%	3.809%
Mortality	Female	Quarters 3-8	0.029%	0.826%	-1.648%	1.591%
Mortality	Male	Years 3-10	0.007%	0.784%	-1.543%	1.530%
Mortality	Female	Quarters 3-8	-0.038%	0.774%	-1.479%	1.555%
Mortality	Male	Years 3-10	0.011%	0.675%	-1.335%	1.312%
Mortality	Female	Ultimate	0.002%	1.412%	-2.772%	2.767%
Mortality	Male	Ultimate	0.018%	1.165%	-2.302%	2.266%

Section 8: R Code

All graduated rates produced under this project were developed with the open-source R software version 4.1.2 (2021-11-01) -- "Bird Hippie". All due care was taken to vet the code as it was developed, and the high quality of the graduated rate fits support a conclusion of the reliability of the code. But any use of the supplied code and associated data sets to produce other analysis and/or rates is done so as the sole responsibility of the user.

There are three sets of code that functionally, 1) produced the rates, 2) performed cross validations and test validations and 3) performed bootstrapping analysis. There is a substantial amount of common code across the three sets. With substantially more time they could have been consolidated into one script but that was not critical to producing the desired project work products.


Rate Fit v3.6 & All Select Analysis v3.75


Cross Val 3.8 & TestFinal 3.95 06222


Bootstrap Rate Fit v3.6 & All Select An

Experienced R coders may find that some of the code could be made more efficient. But generally, speed was not a noticeable limiting factor except for the bootstrapping code which is quite time intensive. Aggregation was explored to speed results but not found to be beneficial. Additionally, while aggregation affected AIC results because they depend on the number of rows, it did not affect the determined regression coefficients and thus had no effect on the rates. As a result, the data was not aggregated as part of producing the rates.

One area that may stand out to experienced R coders is the approach to getting the predicted or graduated rates from a regression run. Attempts to derive those rates with predict functions were not successful so the code was written to take the resulting regression coefficients to produce the rates. In some sense this had about the same degree of efficiency because the predict function requires a dataframe of the parameter inputs, e.g., duration, age at disability and other modifiers (true/false conditions) to enable the function to work. Instead of setting up such a dataframe for a predict function, values were taken as needed and combined with the regression coefficients using first principles of the log transform rate calculation to derive the rates.

Four sets of data are required to run the programs.

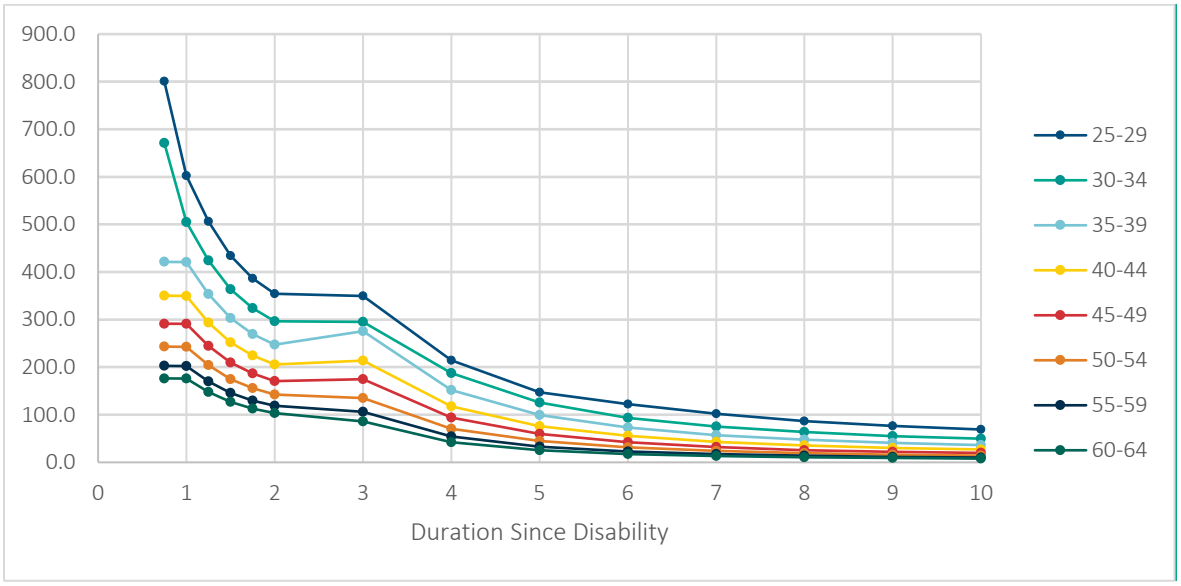
1) Training data for internal use only (contains company codes)
   "M:\Research\Experience Studies - Contractors\RJH\GLW\Train70 061021.csv"
2) Test data for internal use only (contains company codes)
   "M:\Research\Experience Studies - Contractors\RJH\GLW\Test30 061021.csv"
3) GLW2005 Rates – used for predictor inputs for only two of the recovery rate groups – female and male years 3-10.

   
   Recovery GLW2005 Rates.xlsx
4) GLM run input – text file of the regression call that accepts multiple regressions in one run. The practical limit here is nine GLM inputs per run due to space limitations on the summary pdf output. This is a sample file.

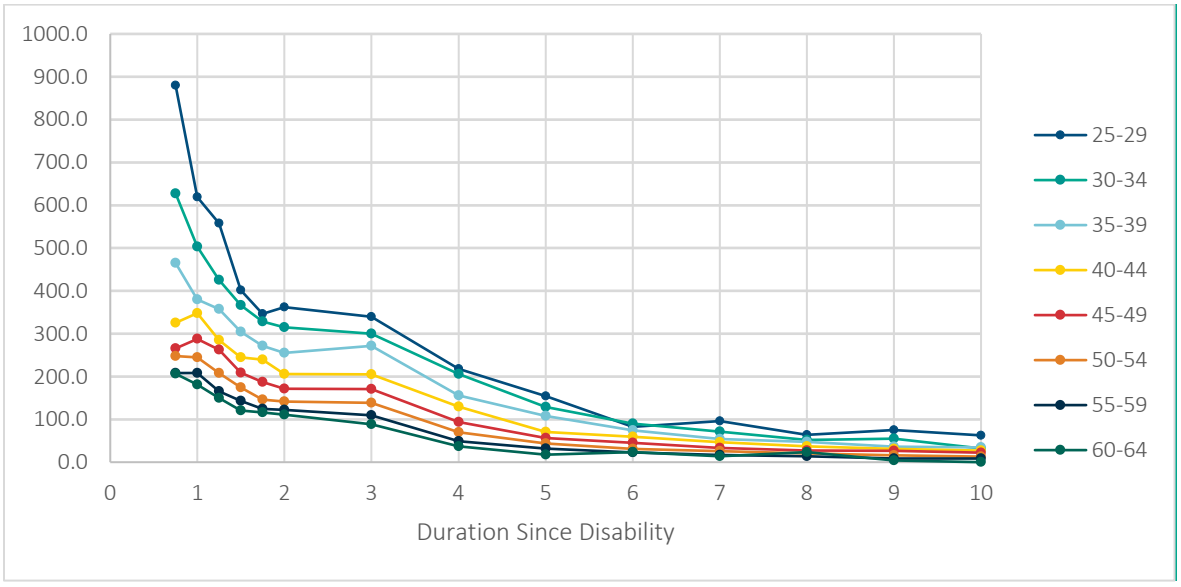
   
   GLM_Run_Input.txt

Appendix A: Graduated vs. Actual Rate Comparisons

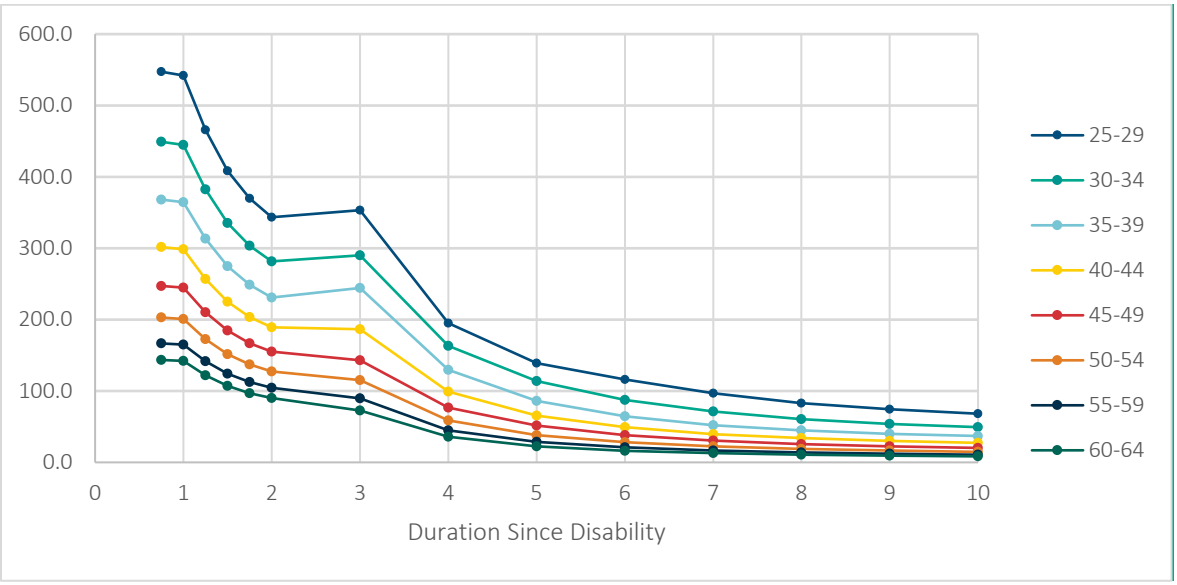
RECOVERY SELECT FEMALE GRADUATED BASE RATES PER 1,000 BY AGE GROUP



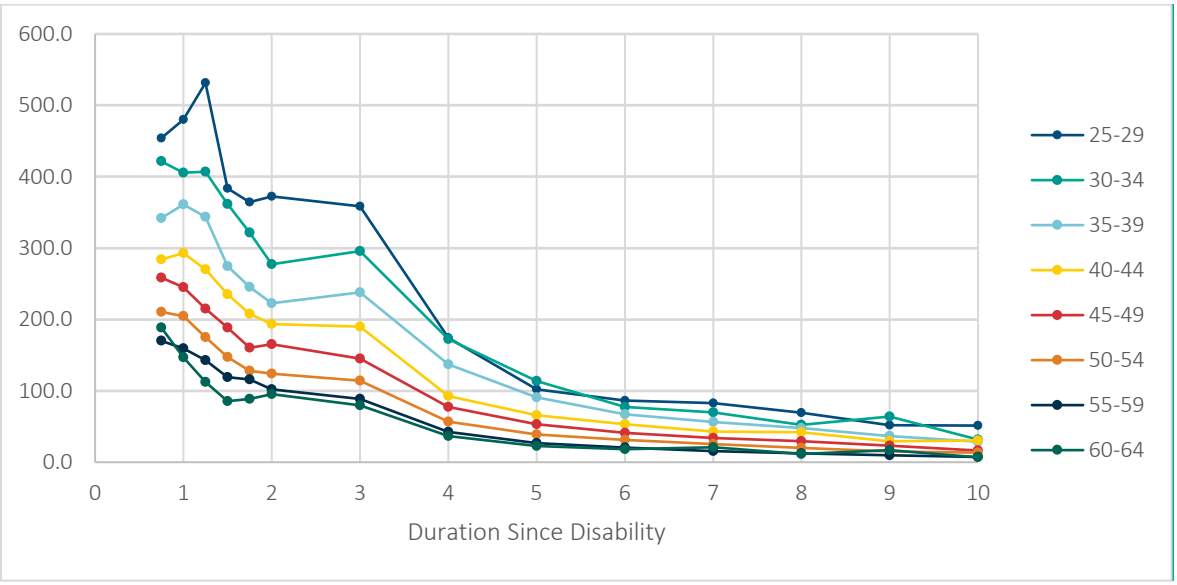
RECOVERY SELECT FEMALE ACTUAL RATES PER 1,000 BY AGE GROUP



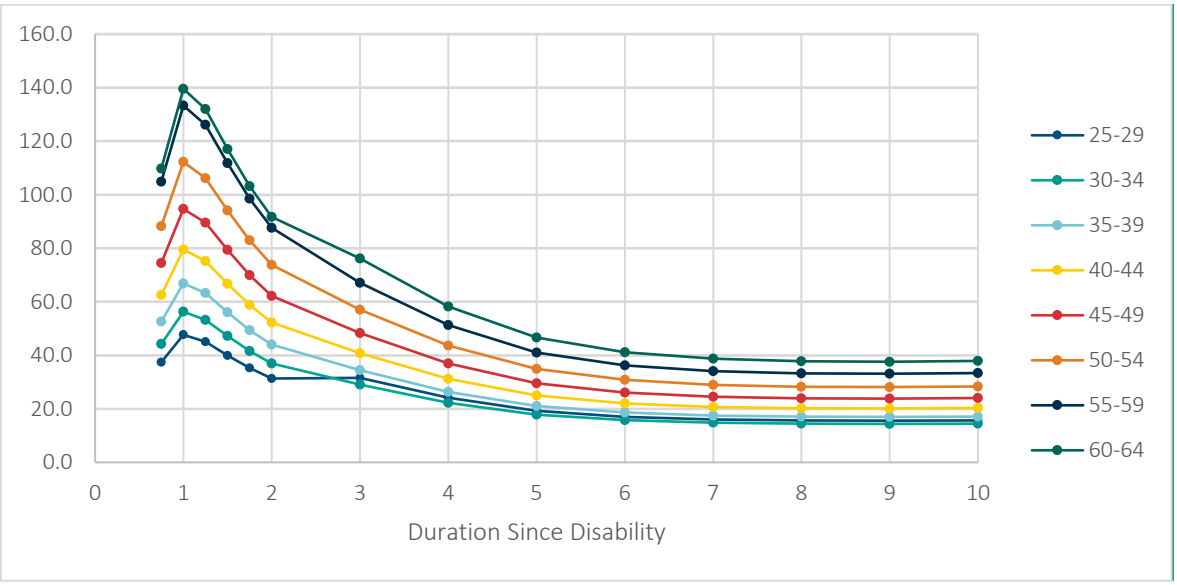
RECOVERY SELECT MALE GRADUATED BASE RATES PER 1,000 BY AGE GROUP



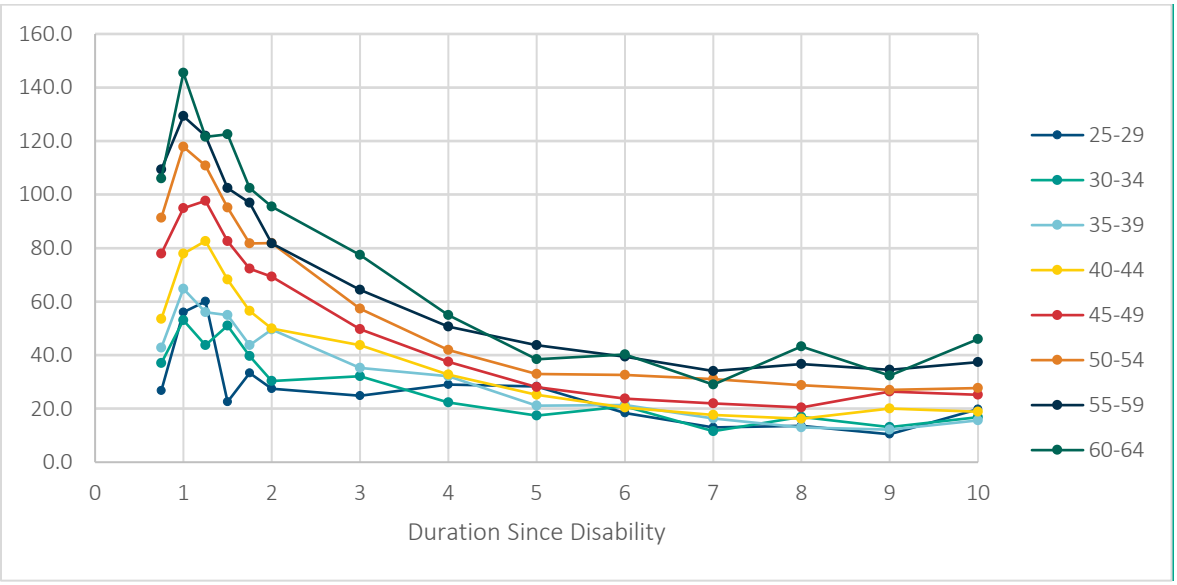
RECOVERY SELECT MALE ACTUAL RATES PER 1,000 BY AGE GROUP



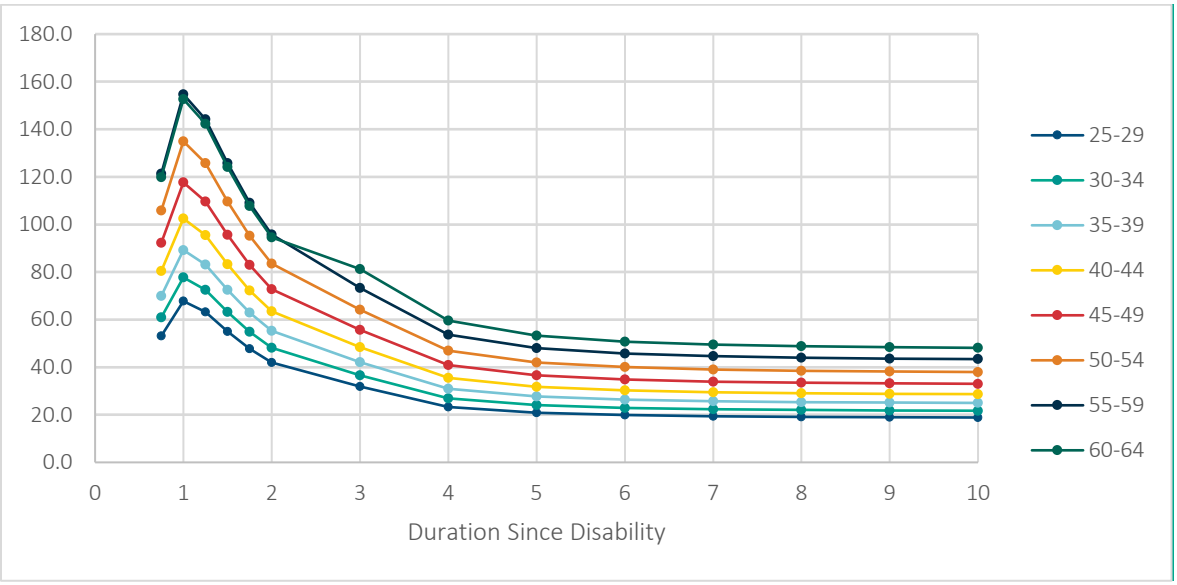
MORTALITY SELECT FEMALE GRADUATED BASE RATES PER 1,000 BY AGE GROUP



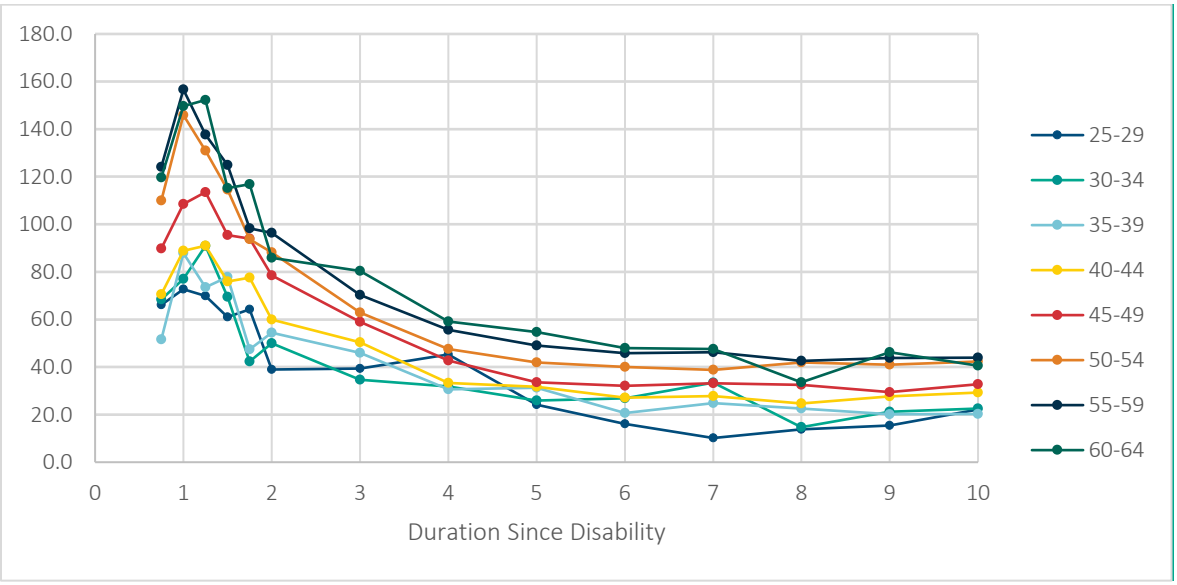
MORTALITY SELECT FEMALE ACTUAL RATES PER 1,000 BY AGE GROUP



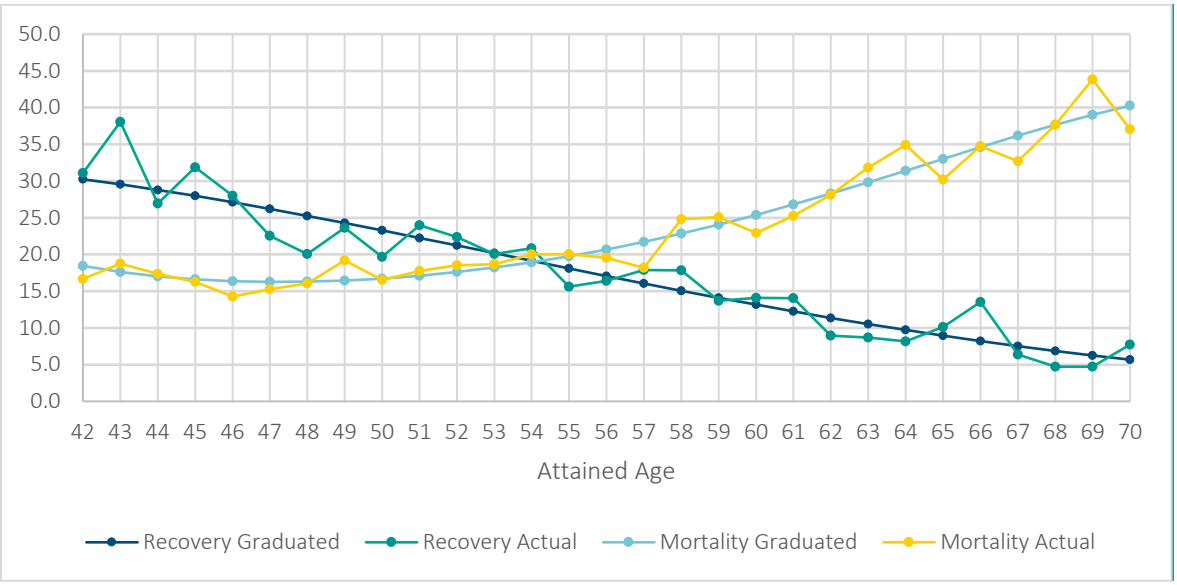
MORTALITY SELECT MALE GRADUATED BASE RATES PER 1,000 BY AGE GROUP



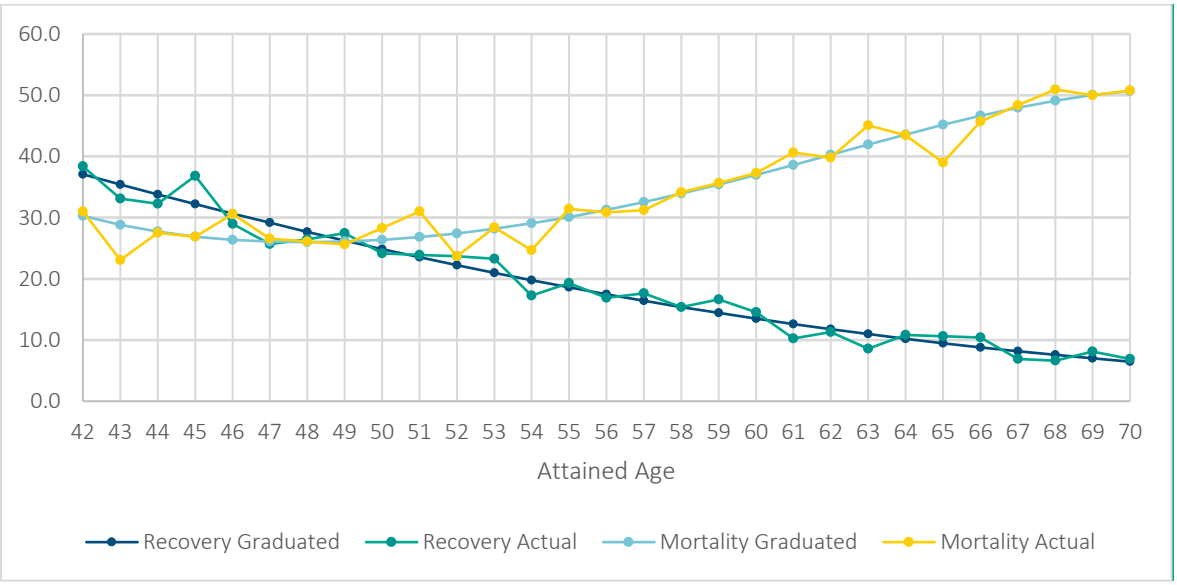
MORTALITY SELECT MALE ACTUAL RATES PER 1,000 BY AGE GROUP



ULTIMATE FEMALE GRADUATED BASE VS. ACTUAL RATES PER 1,000 BY ATTAINED AGE



ULTIMATE MALE GRADUATED BASE VS. ACTUAL RATES PER 1,000 BY ATTAINED AGE



Appendix B: Tabular Base Rates and Diagnosis Adjustments

RECOVERY FEMALE SELECT BASE RATES PER 1,000 BY AGE GROUP AND DURATION

Duration	24-29	30-34	35-39	40-44	45-49	50-54	55-59	60-64
Qtr 3	800.6	671.1	421.3	350.4	291.4	243.3	202.7	176.4
Qtr 4	602.4	505.0	420.6	349.8	290.9	242.9	202.4	176.1
Qtr 5	506.0	424.2	353.3	293.9	244.4	204.1	170.0	147.9
Qtr 6	434.1	363.9	303.1	252.1	209.6	175.0	145.9	126.9
Qtr 7	386.3	323.8	269.7	224.3	186.6	155.8	129.8	112.9
Qtr 8	354.1	296.8	247.2	205.6	171.0	142.8	119.0	103.5
Yr 3	349.8	295.0	275.8	213.5	174.5	134.9	106.5	86.1
Yr 4	214.6	187.4	151.6	117.4	94.2	70.4	54.3	42.4
Yr 5	147.1	125.5	99.8	75.9	59.9	44.7	32.7	25.4
Yr 6	122.2	93.4	72.9	55.5	42.3	31.6	22.7	17.4
Yr 7	102.0	75.3	57.1	43.2	32.0	24.0	17.3	13.2
Yr 8	86.7	63.5	47.4	35.6	25.8	19.2	13.8	10.7
Yr 9	76.5	55.1	40.6	30.4	22.0	16.2	11.7	9.1
Yr 10	69.0	49.8	36.2	27.1	19.8	14.2	10.1	7.9

RECOVERY MALE SELECT BASE RATES PER 1,000 BY AGE GROUP AND DURATION

Duration	24-29	30-34	35-39	40-44	45-49	50-54	55-59	60-64
Qtr 3	547.3	449.2	368.2	301.6	247.2	202.9	166.7	143.5
Qtr 4	542.0	444.9	364.6	298.7	244.8	201.0	165.0	142.1
Qtr 5	466.0	382.5	313.5	256.8	210.5	172.8	141.9	122.2
Qtr 6	408.5	335.3	274.8	225.2	184.5	151.5	124.4	107.1
Qtr 7	369.8	303.5	248.7	203.8	167.0	137.1	112.6	97.0
Qtr 8	343.2	281.7	230.9	189.2	155.0	127.3	104.5	90.0
Yr 3	353.1	290.2	244.2	186.7	142.9	115.2	89.6	72.6
Yr 4	195.1	163.0	129.8	99.2	76.8	58.9	45.1	35.7
Yr 5	139.1	113.7	86.2	65.5	51.6	38.0	28.8	22.7
Yr 6	116.1	87.4	64.7	49.2	38.3	28.4	21.0	16.4
Yr 7	96.7	71.2	52.2	39.7	30.7	22.5	16.7	13.0
Yr 8	83.0	60.7	45.0	34.2	25.6	18.9	13.8	10.9
Yr 9	74.4	53.8	39.9	30.2	22.5	16.5	12.0	9.5
Yr 10	68.2	49.3	37.0	27.3	20.3	14.7	10.7	8.4

MORTALITY FEMALE SELECT BASE RATES PER 1,000 BY AGE GROUP AND DURATION

Duration	24-29	30-34	35-39	40-44	45-49	50-54	55-59	60-64
Qtr 3	37.5	44.3	52.6	62.6	74.5	88.3	104.8	109.7
Qtr 4	47.7	56.3	66.9	79.6	94.7	112.2	133.3	139.5
Qtr 5	45.1	53.2	63.3	75.3	89.6	106.2	126.1	132.0
Qtr 6	40.0	47.2	56.1	66.7	79.4	94.1	111.8	117.1
Qtr 7	35.2	41.6	49.4	58.8	70.0	82.9	98.5	103.1
Qtr 8	31.3	37.0	44.0	52.3	62.2	73.8	87.6	91.7
Yr 3	31.5	29.1	34.4	40.8	48.3	57.0	67.1	76.2
Yr 4	24.1	22.3	26.3	31.2	36.9	43.6	51.3	58.2
Yr 5	19.3	17.8	21.1	25.0	29.5	34.9	41.1	46.6
Yr 6	17.0	15.7	18.6	22.0	26.1	30.8	36.3	41.1
Yr 7	16.0	14.8	17.5	20.7	24.5	29.0	34.1	38.7
Yr 8	15.6	14.4	17.1	20.2	23.9	28.3	33.3	37.7
Yr 9	15.6	14.4	17.0	20.1	23.8	28.2	33.1	37.6
Yr 10	15.7	14.5	17.1	20.3	24.0	28.4	33.4	37.9

MORTALITY MALE SELECT BASE RATES PER 1,000 BY AGE GROUP AND DURATION

Duration	24-29	30-34	35-39	40-44	45-49	50-54	55-59	60-64
Qtr 3	53.2	61.0	70.0	80.4	92.3	105.9	121.4	119.8
Qtr 4	67.8	77.7	89.2	102.5	117.7	134.9	154.7	152.7
Qtr 5	63.2	72.4	83.2	95.5	109.7	125.8	144.2	142.3
Qtr 6	55.1	63.2	72.5	83.3	95.6	109.7	125.7	124.1
Qtr 7	47.8	54.8	62.9	72.3	83.0	95.2	109.1	107.7
Qtr 8	41.9	48.1	55.2	63.4	72.8	83.5	95.8	94.5
Yr 3	31.8	36.7	42.2	48.4	55.7	64.1	73.3	81.3
Yr 4	23.3	26.9	30.9	35.5	40.8	47.0	53.7	59.5
Yr 5	20.9	24.0	27.7	31.8	36.6	42.0	48.1	53.3
Yr 6	19.9	22.9	26.3	30.2	34.8	40.0	45.8	50.7
Yr 7	19.4	22.3	25.7	29.5	34.0	39.0	44.6	49.5
Yr 8	19.1	22.0	25.3	29.1	33.5	38.5	44.0	48.8
Yr 9	19.0	21.8	25.1	28.8	33.2	38.2	43.6	48.4
Yr 10	18.9	21.7	25.0	28.7	33.0	38.0	43.4	48.1

SELECT PERIOD BASE RATE DIAGNOSIS ADJUSTMENTS

Duration	Recovery			Mortality		
	Low	Medium	High	Low Non-Cancer	High Non-Cancer	Cancer
Qtr 3	0.55	1.05	1.60	0.20	0.50	3.65
Qtr 4	0.55	1.15	1.35	0.20	0.50	3.95
Qtr 5	0.60	1.15	1.40	0.20	0.60	4.20
Qtr 6	0.65	1.15	1.35	0.25	0.60	4.50
Qtr 7	0.65	1.15	1.35	0.30	0.70	4.70
Qtr 8	0.75	1.10	1.30	0.30	0.80	4.90
Yr 3	0.65	1.15	1.40	0.40	0.95	4.75
Yr 4	0.60	1.10	1.65	0.50	1.25	4.60
Yr 5	0.70	1.10	1.55	0.60	1.35	4.00
Yr 6	0.70	1.15	1.45	0.65	1.45	3.60
Yr 7	0.70	1.25	1.35	0.70	1.55	3.05
Yr 8	0.75	1.15	1.35	0.70	1.65	2.65
Yr 9	0.70	1.25	1.35	0.75	1.60	2.30
Yr 10	0.70	1.10	1.45	0.75	1.65	2.15

ULTIMATE PERIOD BASE RATE DIAGNOSIS ADJUSTMENTS

Age Group	Recovery			Mortality		
	Low	Medium	High	Low Non-Cancer	High Non-Cancer	Cancer
40-44	0.70	1.40	1.10	0.75	1.90	2.00
45-49	0.70	1.40	1.05	0.85	1.35	1.55
50-54	0.75	1.30	1.10	0.85	1.45	1.70
55-59	0.75	1.25	1.20	0.80	1.40	2.20
60-64	0.85	1.20	1.05	0.80	1.55	1.80
65-69	0.85	1.05	1.20	0.75	1.45	1.95

Appendix C: Base Rate and Diagnosis Adjusted Rate Residuals

RECOVERY RATE FITTING RESIDUALS

			(1)	(2)	(3)	(4)
Rate Basis	Sex	Diagnosis Group	Wtd Residual	Residual % of Actual Rate	Exposure	(1)*(3)
Base Rates						
Select	Male		0.00%	0.00%	657,795.5	(0.0)
	Female		0.00%	0.00%	701,028.2	(0.0)
	Both		0.00%	0.00%	1,358,823.7	(0.0)
Ultimate	Male		0.00%	-0.04%	192,257.6	(1.0)
	Female		0.00%	0.00%	197,849.2	0.0
	Both		0.00%	-0.02%	390,106.8	(1.0)
Diagnosis Adjusted Rates						
Select						
	Female	No Diagnosis	1.85%	21.52%	160,627.6	2,966.9
		Low	-0.37%	-5.43%	216,542.3	(810.8)
		Medium	-0.71%	-4.92%	177,972.8	(1,272.4)
		High	1.02%	6.28%	145,885.5	1,490.5
		Combined	0.34%	3.03%	701,028.2	2,374.3
	Male	No Diagnosis	1.20%	16.85%	159,780.5	1,916.8
		Low	-0.69%	-11.49%	240,071.0	(1,648.4)
		Medium	-0.45%	-3.85%	163,209.0	(737.9)
		High	-0.55%	-3.76%	94,735.0	(524.6)
		Combined	-0.15%	-1.69%	657,795.5	(994.1)
	Both	No Diagnosis	1.52%	19.41%	320,408.1	4,883.7
		Low	-0.54%	-8.40%	456,613.3	(2,459.2)
		Medium	-0.59%	-4.46%	341,181.8	(2,010.3)
		High	0.40%	2.56%	240,620.5	965.9
	Both	Combined	0.10%	1.01%	1,358,823.7	1,380.2
Ultimate						
	Female	No Diagnosis	0.35%	32.47%	88,653.4	307.5
		Low	-0.24%	-17.19%	48,321.1	(115.0)
		Medium	-0.32%	-15.45%	31,713.5	(102.3)
		High	-0.31%	-16.14%	29,161.3	(90.7)
		Combined	0.00%	-0.02%	197,849.2	(0.5)
	Male	No Diagnosis	0.17%	14.24%	95,446.7	165.8
		Low	-0.07%	-5.45%	44,716.1	(29.1)
		Medium	-0.26%	-12.09%	34,200.7	(89.5)
		High	-0.14%	-7.78%	17,894.1	(24.6)
		Combined	0.01%	0.82%	192,257.6	22.6
	Both	No Diagnosis	0.26%	22.41%	169,600.3	436.0
		Low	-0.15%	-11.98%	91,709.9	(142.1)
		Medium	-0.29%	-13.68%	60,901.5	(177.2)
		High	-0.25%	-13.13%	55,689.5	(136.5)
	Both	Combined	0.01%	0.40%	390,106.8	22.1

MORTALITY RATE FITTING RESIDUALS

			(1)	(2)	(3)	(4)
Rate Basis	Sex	Diagnosis Group	Wtd Residual	Residual % of Actual Rate	Exposure	(1)*(3)
Base Rates						
Select	Male		0.00%	0.00%	653,637.2	0.00
	Female		0.00%	0.00%	691,460.7	0.00
	Both		0.00%	0.00%	1,345,097.9	0.0
Ultimate	Male		0.00%	0.00%	193,764.3	0.00%
	Female		0.00%	0.00%	198,631.1	0.00%
	Both		0.00%	0.00%	392,395.5	(0.0)
Diagnosis Adjusted Rates						
Select						
	Female	No Diagnosis	0.29%	6.42%	159,307.5	460.8
		Low Non-Cancer	0.30%	17.84%	389,515.1	1,175.4
		High Non-Cancer	0.17%	3.67%	79,711.2	137.6
		Cancer	-0.31%	-1.16%	62,926.9	(193.5)
		Combined	0.23%	2.05%	691,460.7	1,580.4
	Male	No Diagnosis	0.16%	2.88%	159,344.9	261.4
		Low Non-Cancer	-0.37%	-12.97%	359,014.6	(1,339.0)
		High Non-Cancer	-0.09%	-1.48%	86,426.8	(78.2)
		Cancer	0.79%	2.65%	48,850.9	386.7
		Combined	-0.12%	-1.32%	653,637.2	(769.0)
	Both	No Diagnosis	0.23%	4.44%	318,652.5	722.2
		Low Non-Cancer	-0.02%	-0.97%	748,529.7	(163.5)
		High Non-Cancer	0.04%	0.66%	166,138.0	59.5
		Cancer	0.17%	0.62%	111,777.8	193.3
	Both	Combined	0.060%	1.10%	1,345,097.9	811.4
Ultimate						
	Female	No Diagnosis	0.40%	17.72%	88,998.4	353.4
		Low Non-Cancer	-0.19%	-8.34%	83,396.7	(154.4)
		High Non-Cancer	-0.52%	-12.06%	15,706.4	(82.2)
		Cancer	-0.99%	-16.17%	10,529.7	(104.6)
		Combined	0.01%	0.23%	198,631.1	12.1
	Male	No Diagnosis	0.33%	9.16%	96,149.6	313.9
		Low Non-Cancer	-0.34%	-10.19%	74,185.0	(251.0)
		High Non-Cancer	-0.34%	-5.72%	13,715.1	(46.3)
		Cancer	-0.06%	-0.77%	9,714.7	(5.6)
		Combined	0.01%	0.15%	193,764.3	11.1
	Both	No Diagnosis	0.36%	12.31%	185,148.0	667.4
		Low Non-Cancer	-0.26%	-9.40%	157,581.6	(405.4)
		High Non-Cancer	-0.44%	-8.62%	29,421.5	(128.6)
		Cancer	-0.54%	-8.03%	20,244.4	(110.2)
	Both	Combined	0.01%	0.18%	392,395.5	23.2

Appendix D: GLM Models

RECOVERY MODELS

Sex	Duration	Model
Female	Quarters 3-8	glm(recovery_count ~ avg_group_dis_age + I((1/duration_since_disability)^2) +I((1/duration_since_disability)^3) +I(avg_group_dis_age <= 35 & duration_since_disability <= 0.75) + offset(log(recovery_count_exposed_mixed)),family="poisson",data=df)
Female	Years 3-10	glm(recovery_count ~ avg_group_dis_age + I(1/duration_since_disability) +I((1/duration_since_disability)^2) +I(avg_group_dis_age <= 35 & duration_since_disability == 3) + GLW2005Rate_new + offset(log(recovery_count_exposed)),family="poisson",data=df)
Male	Quarters 3-8	glm(recovery_count ~ avg_group_dis_age + I((1/duration_since_disability)^2) +I((1/duration_since_disability)^3) +offset(log(recovery_count_exposed_mixed)),family="poisson",data=df)
Male	Years 3-10	glm(recovery_count ~ avg_group_dis_age + I(1/duration_since_disability) +I((1/duration_since_disability)^2) +GLW2005Rate_new + offset(log(recovery_count_exposed)),family="poisson",data=df)
Both	Ultimate	glm(recovery_count ~ attained_age + I(attained_age^2) + offset(log(recovery_count_exposed)),family="poisson",data=df)

MORTALITY MODELS

Sex	Duration	Model
Female	Quarters 3-8	glm(death_count ~ avg_group_dis_age + I(1/duration_since_disability) +I((1/duration_since_disability)^2) +I(avg_group_dis_age >= 59) + offset(log(death_count_exposed_mixed)),family="poisson",data=df)
Female	Years 3-10	glm(death_count ~ avg_group_dis_age + I(1/duration_since_disability) +I((1/duration_since_disability)^2) +I((1/duration_since_disability)^3) +I(avg_group_dis_age <= 30) + offset(log(death_count_exposed)),family="poisson",data=df)
Male	Quarters 3-8	glm(death_count ~ avg_group_dis_age + I(1/duration_since_disability) +I((1/duration_since_disability)^2) +I(avg_group_dis_age >= 59) + offset(log(death_count_exposed_mixed)),family="poisson",data=df)
Male	Years 3-10	glm(death_count ~ avg_group_dis_age + I((1/duration_since_disability)^3) + offset(log(death_count_exposed)),family="poisson",data=df)
Both	Ultimate	glm(death_count ~ attained_age + I(attained_age^2) + I(attained_age^3) + offset(log(death_count_exposed)),family="poisson",data=df)

Endnotes

[1] Gareth James, Daniela Witten, Trevor Hastie and Robert Tibshirani, *An Introduction to Statistical Learning with Applications in R* (Springer Science+Business Media New York 2013 (Corrected at 4 printing 2014)), 1,26.
[2] *Mike Marin* *9.11 Poisson Regression: Model Assumptions - YouTube* (https://www.youtube.com/hashtag/marinstatslectures University of British Columbia) 2021
[3] John Fox, *Bootstrapping Regression Models Appendix to an R and S-PLUS Companion to Applied Regression* 2002 https://artowen.su.domains/courses/305-1314/FoxOnBootingRegInR.pdf

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Society of Actuaries Research Institute
475 N. Martingale Road, Suite 600
Schaumburg, Illinois 60173
www.SOA.org