# Flooring the GEMS interest rate model

A SHADOW-RATE APPROXIMATION

*The goal is a simple floor that does not sacrifice the arbitrage-free nature of the model.*

*A good approximation appears possible.*

PREPARED BY STEPHEN J. STROMMEN FSA, CERA, MAAA

DECEMBER 2021

# What is a shadow-rate model?

It is an adjusted version of arbitrage-free interest rate model. The adjustment is that downward movements of the short-term risk-free rate are constrained when they go below some level such as zero.

Example: Use GEMS as-is except constrain downward movement of the short-term rate.

- For example: *Constrained rate = greater of (unconstrained rate, 0.2 x unconstrained rate)*

Shadow-rate model uses GEMS native process to project the state variables but uses simulation of the constrained r(t) to generate yield curves.

## Problem: Completing the yield curve

# How to complete the yield curve

Price of a zero-coupon bond in an arbitrage-free model:

$$P_T = E\left[exp\left(-\int_0^T r_t dt\right)\right]$$

Stochastic paths or r(t) are based on the risk-neutral parameters.

There is a formula for that price in GEMS.

In a shadow-rate model it must be determined using constrained of paths of r(t).

# Research study undertaken

- Generate a large sample of both yield  curves (native GEMS and constrained).
  - Each curve starts with specific values for the state variables.
  - Do this for an array of different values for the state variables.
  - Each pair of yield curves based on specific state variable values is a “case”
- Collect a data set of cases where the constrained and un-constrained curves differ by at least 0.05%.
- *Try to find a relationship between the two curves that can be used to convert the un-constrained curve to the constrained curve without requiring simulation.*

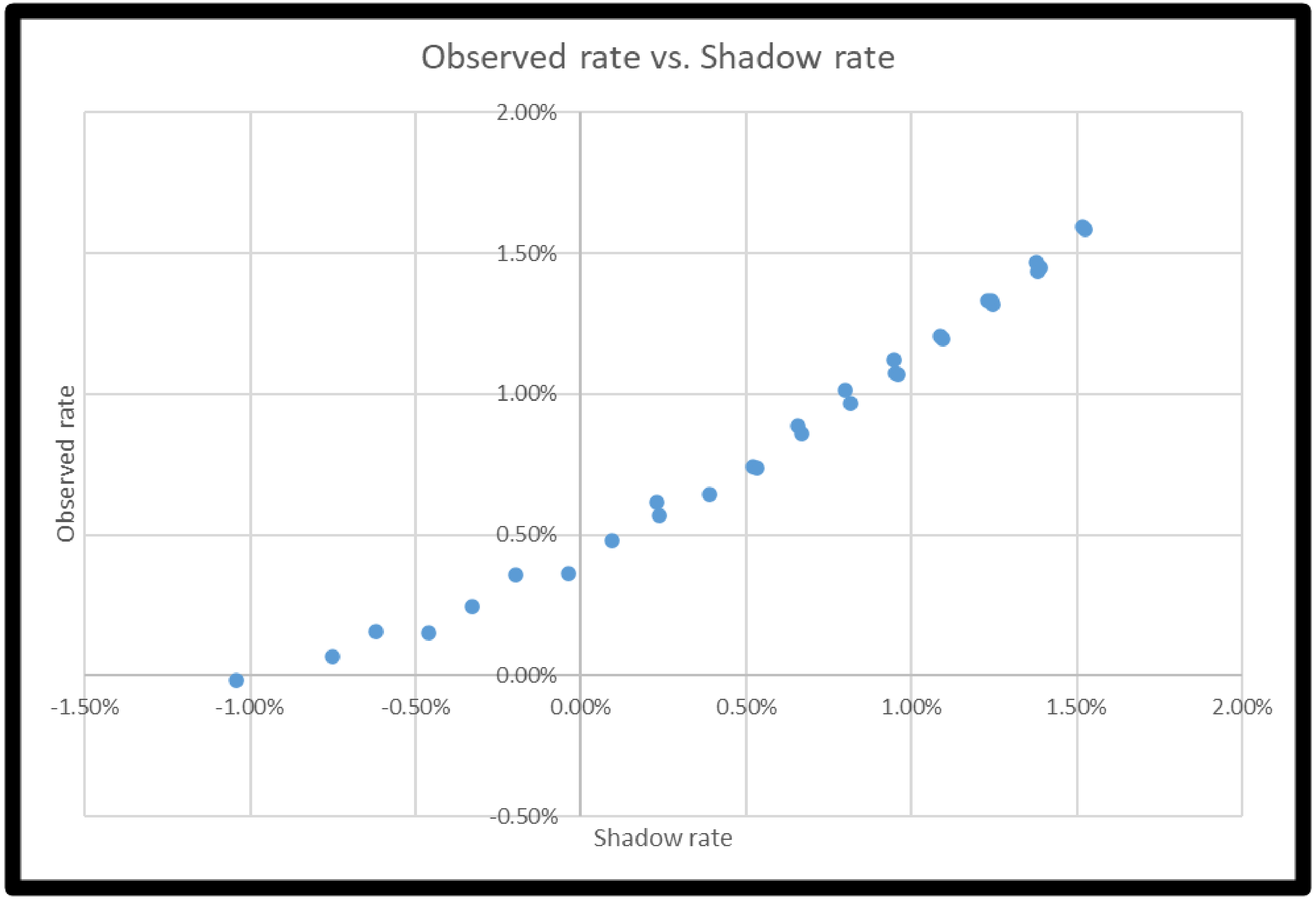
# Comparing curves Constrained vs. Un-constrained

Create X-Y chart of spot rates for a specific term to maturity.

Chart at right is for 2yr maturity.

X = Unconstrained (shadow) rate

Y = Constrained (observed) rate



# Perform linear regression to obtain:

Z = Zero intercept

S = Slope

Get unique Z and S for each term to maturity.

## Using Z and S:

Conversion affects only rates $< Z / (1 – S)$

Observed rate is the constrained rate (the floored rate)

Shadow rate is the un-constrained rate (directly from GEMS)

Observed rate = Z + (Shadow rate)*S

Shadow rate = (Observed rate – Z) / S

# Sample conversion table

Based on constrained rate being 20% of un-constrained rate when below zero.

| Maturity | Slope | Intercept | Applies Below |
|---|---|---|---|
| 0.25 | 0.342385 | 0.13% | 0.19% |
| 1 | 0.547097 | 0.41% | 0.91% |
| 2 | 0.661159 | 0.48% | 1.42% |
| 3 | 0.729228 | 0.49% | 1.81% |
| 4 | 0.771861 | 0.48% | 2.12% |
| 5 | 0.79522 | 0.48% | 2.34% |
| 6 | 0.820546 | 0.46% | 2.58% |
| 7 | 0.829729 | 0.46% | 2.71% |
| 8 | 0.841513 | 0.45% | 2.86% |
| 9 | 0.850308 | 0.45% | 2.99% |
| 10 | 0.858661 | 0.44% | 3.12% |
| 11 | 0.86037 | 0.44% | 3.18% |
| 12 | 0.866498 | 0.44% | 3.29% |
| 13 | 0.87203 | 0.43% | 3.39% |
| 14 | 0.87684 | 0.43% | 3.48% |
| 15 | 0.881521 | 0.42% | 3.57% |
| 16 | 0.886208 | 0.42% | 3.66% |
| 17 | 0.890237 | 0.41% | 3.74% |
| 18 | 0.894092 | 0.40% | 3.81% |
| 19 | 0.896726 | 0.40% | 3.88% |
| 20 | 0.914074 | 0.36% | 4.19% |
| 21 | 0.915533 | 0.36% | 4.24% |
| 22 | 0.917415 | 0.36% | 4.31% |
| 23 | 0.918795 | 0.35% | 4.37% |
| 24 | 0.919672 | 0.35% | 4.42% |
| 25 | 0.921638 | 0.35% | 4.49% |
| 26 | 0.926934 | 0.34% | 4.65% |
| 27 | 0.928018 | 0.34% | 4.71% |
| 28 | 0.929966 | 0.34% | 4.80% |
| 29 | 0.930592 | 0.34% | 4.84% |
| 30 | 0.933308 | 0.33% | 4.96% |

# Comparison of curves

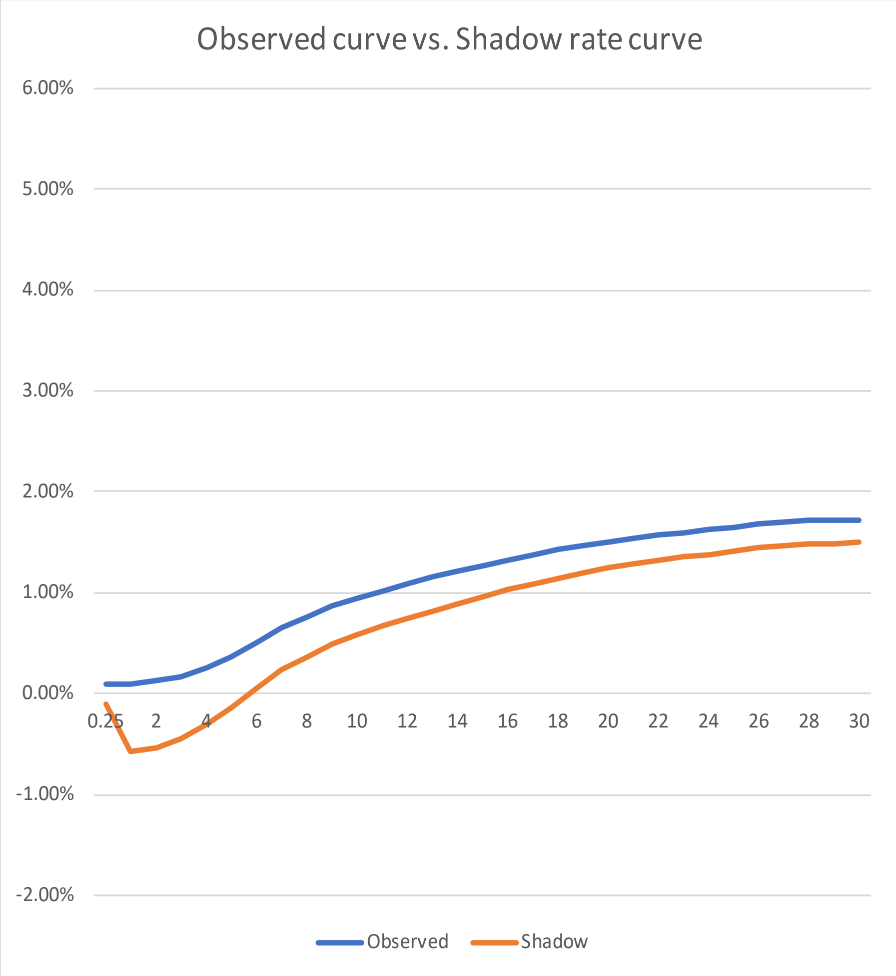
Observed curve reflects floor.

Shadow rate curve does not.

| | Observed | Shadow |
|---|---|---|
| 0.25 | 0.09% | -0.11% |
| 1 | 0.10% | -0.57% |
| 2 | 0.13% | -0.53% |
| 3 | 0.17% | -0.44% |
| 4 | 0.25% | -0.30% |
| 5 | 0.36% | -0.15% |
| 6 | 0.51% | 0.05% |
| 7 | 0.66% | 0.23% |
| 8 | 0.77% | 0.37% |
| 9 | 0.86% | 0.49% |
| 10 | 0.94% | 0.59% |
| 11 | 1.02% | 0.66% |
| 12 | 1.09% | 0.75% |
| 13 | 1.15% | 0.82% |
| 14 | 1.21% | 0.90% |
| 15 | 1.27% | 0.97% |
| 16 | 1.33% | 1.03% |
| 17 | 1.38% | 1.09% |
| 18 | 1.43% | 1.14% |
| 19 | 1.47% | 1.19% |
| 20 | 1.50% | 1.25% |
| 21 | 1.53% | 1.28% |
| 22 | 1.57% | 1.32% |
| 23 | 1.60% | 1.35% |
| 24 | 1.62% | 1.38% |
| 25 | 1.65% | 1.41% |
| 26 | 1.67% | 1.44% |
| 27 | 1.69% | 1.46% |
| 28 | 1.71% | 1.48% |
| 29 | 1.72% | 1.49% |
| 30 | 1.73% | 1.50% |

Date(YYYYMM)

202012



# Scenario generation process

Includes change to how the initial state variables are determined.

1. Convert the observed starting curve to a shadow curve.
2. Fit the state variables to the shadow curve because GEMS is used as a shadow-rate model.
3. Generate the scenarios. GEMS produces the shadow curves.
4. Convert all the generated shadow curves to observed curves for release.

# Next steps

… if the concept is accepted …

Conversion table depends on the definition of the floor and on values of the GEMS parameters.

Next steps include:

- Specify the GEMS parameters
- Define 2 or 3 versions of a floor (e.g. 20%, 50%)
- Create conversion table for each floor (*)
- For each floor:
    - Calculate the initial shadow curve
    - Fit GEMS state variables to that curve
    - Generate shadow-rate scenarios with GEMS
    - Apply the conversion
    - Review the resulting scenarios

"Low-for-long" measure is affected by the floor.  This may indirectly influence choice of GEMS parameter values.

(*) Maybe enhance process to use more robust sample of yield curves