An Approach for Stratifying Economic Scenarios on Both Interest Rates and Equity Returns

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The current American Academy of Actuaries (Academy) Economic Scenario Generator (ESG) has an embedded "Scenario Picker" that is set up to stratify scenarios based on a measure of interest rate levels. Older versions operated on a standalone basis, rather than being embedded in the ESG. These versions allowed the flexibility to choose an equity return element to use in stratification. Thus, the Academy has provided the ability to stratify on either interest rate levels or equity returns, but not both simultaneously.

This report describes an approach for such a "two-dimensional" stratification. The stratification process could be applied to any set of scenarios, such as those generated by the Conning ESG selected by the NAIC to replace the Academy ESG. Conning has already implemented the current Academy stratification process.

Description of Scenario Stratification Process

A January 13, 2006 Academy report, "Construction and Use of Pre-packaged Scenarios to Support the Determination of Regulatory Risk-Based Capital Requirements for Variable Annuities and Similar Products", describes a "Scenario Picking Tool" and methodology, on page 22. The version of the tool mentioned in the report, "Scenario Picking Tool (AAA LCAS C3 Phase II RBC) v7 Locked.xls", enables the user to stratify scenarios based on a selected "Variable Account Asset Class", most commonly the Diversified Large Cap U.S. Equity Returns, which represent the S&P 500 Total Return Index. The updated version of the tool is "ESWG scenario picking tool v2.1.xls", available on the Academy website in a zip file entitled "2009_ESWG_tool.zip". This version allows stratification based on the 1-year or 20-year Treasury rates.

In order to stratify on interest rates and equity returns at the same time, percentiles of measures of both risk elements need to be considered simultaneously. The two dimensional process uses a "van der Corput" (VDC) low discrepancy sequence (LDS) to select pairs of percentiles that are uncorrelated on average but ensure covering the full range of combinations of percentiles for each element (interest rates or equity returns). This process was originally researched, developed and implemented in the 1990's by Jeff Leitz[i], an actuary with Travelers at the time.

"Significance" values, as defined in the 2006 report, are calculated for the full 10,000 scenarios for both Interest Rates and Equity Returns, and the percentiles are then calculated. Thus each of the 10,000 scenarios is represented by a point (x, y), where x is the Interest Rate significance value percentile, and y is the Equity Return significance value percentile.

The "discrepancy" measure refers to the difference between the expected number of points and the actual number of points in all combinations of an arbitrarily small subset of the unit square covering zero to 100% (i.e. zero to one) for the two elements, Interest Rate and Equity Return percentiles. This low discrepancy feature is especially important to producing more even distributions of scenario metrics for small numbers of stratified scenarios, versus random number selection.

Details of Scenario Stratification Process

For each VDC pair (x,y), the sequence of x's is simply mid-points of equally divided intervals of (0,1). These will correspond to the percentiles of interest rate scenarios ranked by their Significance Measures, from low to high. The VDC sequence of y's is constructed by dividing the interval (0,1), level by level with base Z, and performing a mid-point shift similarly to x, as illustrated below. When looking at the final stratified scenarios, there are scenarios with both high positive correlations, between interest rate levels and equity returns, and high negative correlations. The distribution of correlations looks quite realistic when compared to historical observations, with a somewhat wider range.

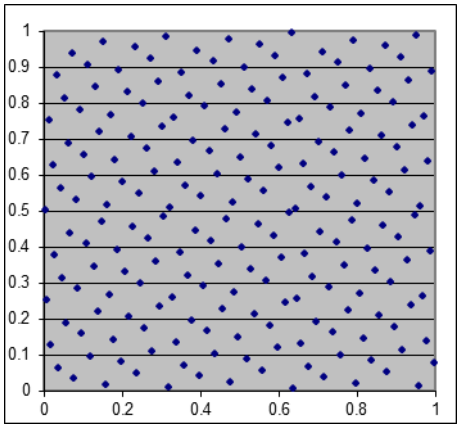
The first set of y contains Z numbers, which will be the lower boundaries of Z equally divided intervals on (0,1), and then adjusted up by a mid-point adjustment to x. At the second level, each interval is further divided into Z smaller intervals, and the 2nd to Z-th set of y values will be the lower bounds of these Z smaller intervals, with the same mid-point adjustment. If a third level is needed, each interval is further divided into Z sub-intervals, until the desired number of scenarios is reached. The number of possible points in each level will increase exponentially.

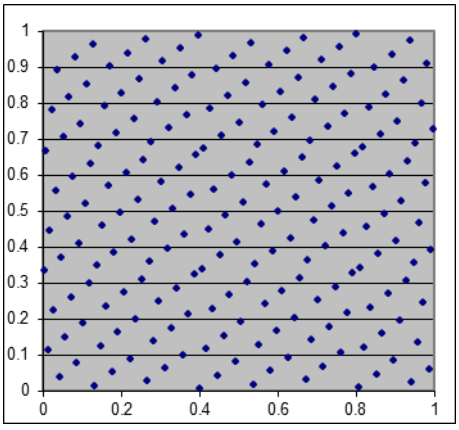
For example, to create a 200 scenario subset using base 3, x = (0.0025, 0.0075, 0.0125, 0.0175, 0.0225…). The mid-point adjustment is 0.0025. The first level produces 2 values of y (first point is discarded), which will be adjusted up by 0.0025 from 1/3 and 2/3, to 0.3358 and 0.6692. The second level produces 6 values of y, which will be adjusted up 0.0025 from 1/9, 2/9, 4/9, 5/9, 7/9 and 8/9, to 0.1136, 0.4469, 0.7803, 0.2247, 0.5581 and 0.8914. Similarly, the third level will be points adjusted up 0.0025 from 1/27, 2/27, 4/27, 5/27, 7/27…, and repeat this process until we totally have 200 points. With base 3, five levels are needed to create 200 points.

Because the resulting VDC sequence of y's does not follow the desired pattern of percentiles (as described by x), these y's are then adjusted one by one to the closest desired percentile, avoiding percentile duplication at the same time. Big adjustments are likely for last few y's for duplication avoidance. We divide all y's into 4 equal sections to do the adjustments circularly, so as to avoid big adjustments clustered together.

For a 200 scenario subset, the y's need to be adjusted in the following order: 51, 101, 151, 1, 52, 102, 152, 2 … 100, 150, 200, 50. Continuing the numerical example above, the adjusted first few y's will be 0.3375, 0.6675, 0.1125, 0.4475, 0.7825, 0.2225, 0.5575, 0.8925…

The graphs below show the base 2 (left) and base 3 (right) VDC points, for a 200 scenario subset. We choose to use base 2 for scenario generation based on visual selection, as the base 3 graph appears to have more points along diagonal lines, and points on any line will tend to have higher discrepancy than more scattered points.





Percentile Adjustment of VDC points

One shortcoming in the VDC stratification process is that the y-axis of the VDC points may not fall on the exact percentiles we want to produce. For example, when producing 200 scenarios, we divide (0,1) to 200 intervals, and would like each scenario to fall right in the middle of each interval. While we can increase x value of points by the percentile we specified exactly, the value of y purely depends on the base we choose. Thus, we will adjust the y value to the closest exact percentile.

To adjust the y values to exact percentiles, we need to loop through all the VDC points, and for each selected y, the closest percentile will be assigned as the adjusted y. This process does not allow any percentile to be selected twice, to ensure the entire distribution is covered. It is possible that the adjustment is large for the last few points. For CTE purposes, when tail scenarios are important, the process starts and flips from both tails and ends in the middle.

The stratification is performed by calculating the distance from each VDC point to each of the 10,000 scenario points. The scenario with shortest distance is selected as the stratified scenario. Repeat the same process for all VDC points, and a subset of scenarios is created.

In rare situations, it is possible that one scenario, out of the total 10,000 scenarios, has the closest distance to two VDC points. If that occurs, this scenario would be selected twice in the stratification process. The duplication does not mean it is a bad stratification; however, the stratification process will keep a record of how many, and which scenarios are selected multiple times.

Calibration Results for 200 and 1,000 scenario sets

The June 2005 Academy report, "Recommended Approach for Setting Regulatory Risk-Based Capital Requirements for Variable Annuities and Similar Products" contains calibration criteria for equity returns. The December 2008 Academy report, "Report from the American Academy of Actuaries' Economic Scenario Work Group To Life Risk Based Capital Task Force and Life and Health Actuarial Task Force" describes recommended calibration criteria for interest rates.

The graphs in Appendix A show calibration results using these criteria, for both interest rates and equity returns, as of 3/31/2020. The recommended criteria have been extended to include every month of the projections. This enables the calculation and display of percentages of the time that calibration

criteria are met. As always, sound actuarial judgment must be applied to determine whether scenarios may still be used if there are failures of the criteria. However, some minor failures, in conjunction with high passing percentages, would likely be acceptable. Conversely, a scenario set that passes the specific calibration criteria may still be problematic, if the percentages of the time that calibration criteria are met is low. Calibration criteria between the current calibration points are based on levelized returns over the intervening time period.

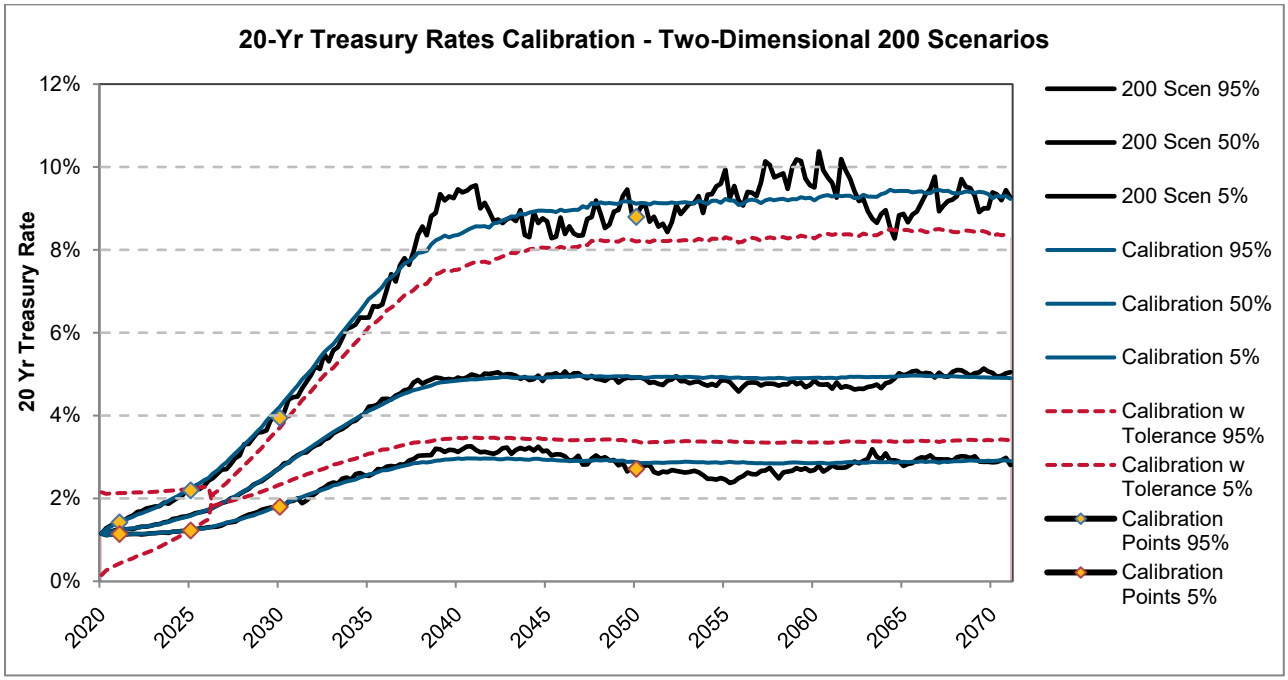
Appendix A – C-3 Phase 2 - Two-dimensional Stratified Scenario Improvement Summary and Calibration Exhibits

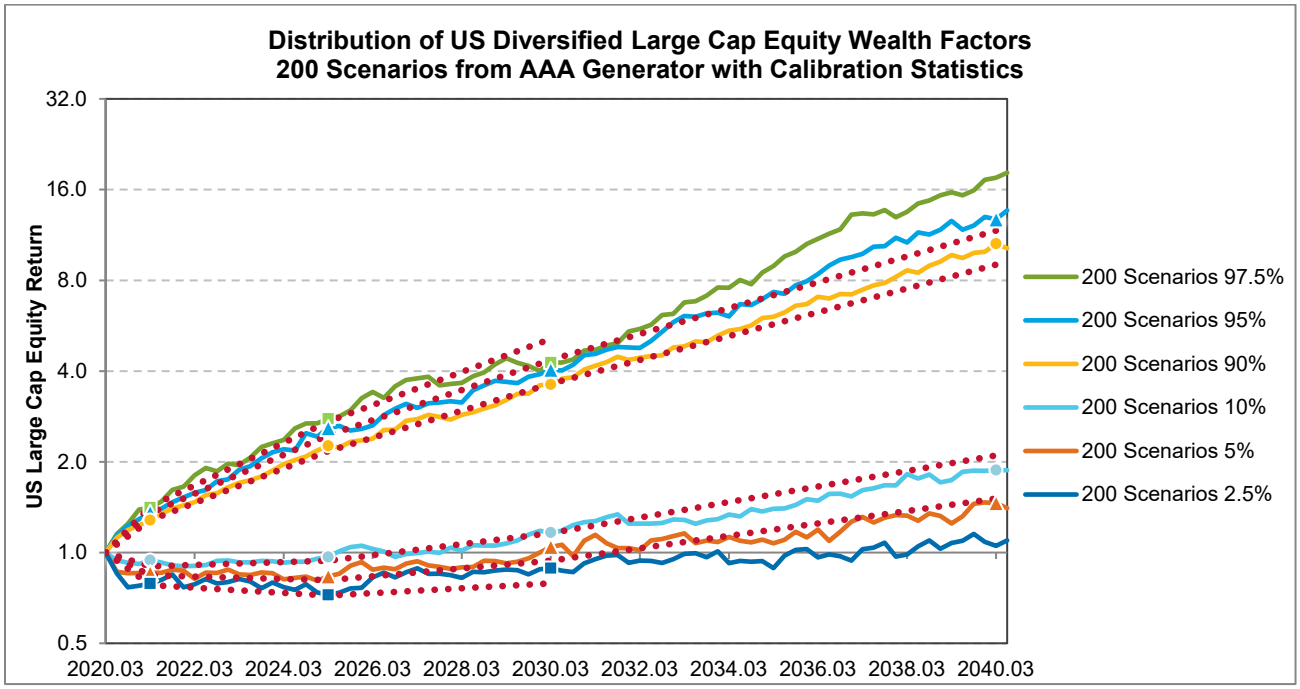
The table below summarizes the equity scenario calibration results for 200 Two-Dimensional stratified scenarios compared with 200 scenarios selected using Interest Only stratification. The Two-Dimensional stratification improved the calibration results significantly.

		Point-In-Time Failures		Average Failures		Average Pass < 50%*	
		Total	Low Side	Total	Low Side	Total	Low Side
Interest Only	Points	11	4	6	2	7	2
	Average Magnitude	1.38%	1.25%	0.16%	0.25%	36.6%	36.8%
Two-dimensional	Points	9	3	2	0	2	0
	Average Magnitude	0.69%	0.11%	0.34%	0.00%	30.9%	0.0%

** Average magnitude for Average Pass Rate < 50% is the average pass rate of these failure points.*

The following exhibits show 200 scenario calibration results, for both interest rate and equity returns, as of 3/31/2020.





[i] Jeffrey A. Leitz, FSA, CERA pioneered multidimensional scenario stratification using low-discrepancy point sequences in 1998. Leitz implemented this method while working at Citigroup's Travelers Life and Annuity Company. Later, as a consulting actuary at Tillinghast Towers Perrin, Leitz played a key role in developing C3 Phase II RBC for the NAIC. Additionally, Leitz founded and managed Walbridge Capital's quantitative beta-neural equity hedge fund from 2008 to 2013, employing a 3D stratification approach for security selection and risk management.