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Integrated Determination of Insurer Capital, Investment and Reinsurance Strategy

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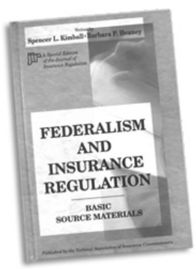

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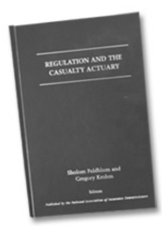

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Integrated Determination of Insurer Capital, Investment and Reinsurance Strategy

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Abstract

Based on the criteria of the Swiss Solvency Test, Solvency II, RBC and minimizing total frictional cost, we establish integrated models to determine the adjustment capital required, investment strategy and reinsurance strategy by numerically analyzing the effect of several important parameters. Results illustrate that when the cost of reinsurance is low or the frictional cost of capital is high, reinsurance is especially attractive as an effective instrument for capital management. However, when the cost of reinsurance is high or the frictional cost of capital is low, capital can partly or fully substitute for reinsurance. Furthermore, in most cases, setting the regulatory capital level either by Solvency II or the Swiss Solvency Test leads to greater prudence than determining insurer capital level by minimizing total frictional cost, except when the cost of capital is very low.

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Introduction

Insurers use various strategies, methods and tools to manage risk, including reinsurance, increasing the amount of capital held and optimizing the investment portfolio. In this light, insurer risk management may be viewed as a system engineering project whereby managers assess the costs and benefits of various financial tools to determine optimal risk management strategies to achieve overall profitability and financial strength.

While capital provides a safety net that helps an insurer reduce the likelihood of financial distress, increasing the amount of capital held is costly (Staking and Babbel, 1995). Developments in global insurance regulation are aimed at risk-based supervision that accounts for all financial, insurance and business risks, particularly asset and liability risks. For example, the Swiss Solvency Test (SST) proposes the concept of target capital.[1] Solvency II, the European Union (EU) framework for prudential regulation of insurers, presents the concept of the solvency capital requirement (SCR).[2] In the U.S., necessary economic capital is defined by the NAIC in terms of an insurer's RBC. Eling, Schmeiser and Schmit (2007), Eling and Holzmuller (2008), Holzmuller (2009), Cummins and Phillips (2009), and Gatzert and Wesker (2012) analyze and compare the recent developments in global solvency regulation.

Dhaene et al. (2003) discuss the determination of optimal capital by minimizing the capital cost above risk-free interest and insolvency cost. Chandra and Sherris (2006) note that minimizing frictional cost of capital produces an optimal capital level based on value at risk (VaR) at much lower levels than observed.[3] They established single-period and multiple-period optimization models under the assumption that the return on insurer assets is deterministic. For a discussion of other research addressing capital and capital allocation, see Mao and Ostaszewski (2010).

1. As discussed in Luder (2005), the Swiss Solvency Test (SST) is a stochastic risk model that includes scenarios for market risk, insurance risk and credit risk in order to determine target capital.

2. Solvency II aims to be a forward-looking, risk-sensitive regulatory structure, focusing on capital adequacy, governance and overall risk management through a total balance sheet approach. Solvency II further develops insurance regulation, similar to developments in bank regulation in Basel II.

3. Frictional cost of capital is defined as taxation and investment cost on assets backing the required capital over the projected lifetime of underlying risk. See: Smith (2010) and *www.pwc.com/gx/en/actuarial-insurance-services/pdf/european-insurance-cfo-forum-mcev.pdf*.

Table 1:
Definition of Variables

$A(t)$	Assets of an insurer at time t
$L(t)$	Liabilities of an insurer at time t
X_0	Amount of initial capital
$X(t)$	Amount of surplus at time t
$K(t)$	Amount of adjustment capital required at time t
c_c	Ratio of frictional capital cost
C_a	The adjustment cost
π	The portion of investment in risky asset
r	Risk-free interest rate
$a(t)$	Reinsurance retention ratio (proportion not reinsured) at time t
η	Cost of reinsurance for exposure unit
P_t	Premium at time t
P_{0t}	Claim loss at time t
σ_1	Volatility of risky asset
σ_2	Volatility of claim loss (diffusion coefficient)
c_f	The ratio of total bankruptcy cost to firm value (also called the coefficient of bankruptcy cost)

In this paper, we extend the work in Chandra and Sherris (2006) by establishing a multi-period optimization model that jointly considers the effect of capital level, reinsurance and investment strategy. We establish a stochastic objective function that minimizes the sum of the frictional capital costs, the reinsurance cost and the cost of financial distress to simultaneously determine optimal capital, reinsurance and investment strategy. We consider the structure of assets and liabilities of property-liability insurers with a numerical analysis of the impact of several important parameters on the optimal capital level, reinsurance and investment strategy. We discuss the different models of determining the necessary economic capital known as target capital under the SST, the SCR under Solvency II and the RBC under the U.S. method (NAIC). We use an example to facilitate the discussions.

Model Based on Minimizing the Friction Cost

The Optimization Model with No Constraint of Insolvency

Our optimization model deals with determining the insurer's optimal capital level. We consider the relationship between bankruptcy cost, frictional capital cost, the expected total frictional cost (the difference between these first two costs) and the amount of adjustment capital required. It is important for insurers to determine the optimal amount of capital to hold, since capital is costly and holding higher levels of capital increases its frictional cost. Holding higher levels of capital generally decreases the probability of insolvency, with a resulting decrease in the expected bankruptcy cost. However, higher levels of capital will increase the frictional cost of capital. In practice, the insurer decides whether to raise or shed some capital (which we refer to as adjustment in capital required) based on the determined optimal capital level. Thus, the insurer faces a trade-off for which the insurer needs to determine the optimal level of capital to hold.

While reinsurance allows insurers to mitigate underwriting risks, Fier, McCullough and Carson (2013) provide empirical evidence that insurers also use reinsurance to adjust capital across affiliated firms within internal capital markets. Beyond the use of reinsurance, effective investment strategy also can help in hedging underwriting risks. In what follows, we will take the approach of integrating risk management and capital management to establish an integrated model for the simultaneous determination of the optimal capital level, reinsurance and investment strategy. We assume proportional reinsurance and one risky asset invested by the insurer, with variables as defined in Table 1.

The expected total frictional cost at time t, $FC_t, t=1,2,\ldots T$ is defined as the sum of the reinsurance cost, the frictional cost of capital and the expected cost of bankruptcy[4] at time t, with $t = 1,2,\ldots T$, that is:

$$FC_t = \sum_{j=1}^{t} P_j(1-a)\eta + c_c E\left(X(t) \middle| X(t) > 0\right) - c_f E\left(X(t) \middle| X(t) < 0\right) + C_a \tag{1}$$

where the surplus $X(t)$ is also called RBC (Schmeiser and Siegel, 2013).

4. Bankruptcy costs can be defined broadly as either direct bankruptcy costs or indirect bankruptcy costs. Direct bankruptcy costs are those explicit costs paid by the debtor in the reorganization/liquidation process, including legal, accounting, filing and other administrative costs related to the liquidation of the firm’s assets. Indirect bankruptcy costs are the opportunity costs of lost management energies, which could lead to lost sales, lost profits, higher cost of credit, and/or possibly the inability of the enterprise to obtain credit or issue securities to finance new opportunities. (See Chandra and Sherris, 2006.)

By establishing the stochastic objective function of minimizing the sum of the reinsurance cost, the frictional cost of capital and the expected cost of bankruptcy, we can find the optimal portion of risky asset to invest, the optimal proportion of reinsurance, the optimal capital level and, therefore, the optimal risk-based capital.[5]

The objective function is:

$$\min \quad FC_t = \sum_{j=1}^{t} P_j(1-a)\eta + c_c K(t) - c_f E\big(X(t)/X(t)<0\big) + C_a \quad , \tag{2}$$

where $K(t)$ is the capital needed to be raised or to be reduced at time t, and we refer to it as the adjustment capital required. C_a, ($C_a \geq 0$) is the adjustment cost associated with raising or shedding external capital.[6] Assume that the surplus of property-liability insurers satisfies the following stochastic differential equation:

$$\begin{aligned} dX(t) &= d\big(A(t) - L(t)\big) = \\ &= \Big(K(t) + a(t)P_t + \pi X(t)(\mu - r) + X(t)r - \big((1-a(t))\eta + a(t)\big)P_{0t} - K(t)r_c - C_a\Big)dt \\ &\quad + \pi X(t)\sigma_1 dW_1 + a(t)\sigma_2 dW_2 \end{aligned} \tag{3}$$

with the boundary condition $X(0) = x$, where W_1 is a geometric Brownian motion with drift of μ and volatility of σ_1, W_2 is a diffusion process with diffusion coefficient of σ_2, W_1 and W_2 are independent of each other, and π is the portion of investment in risky assets. Substituting equation (3) into equation (2), we can obtain the optimal solutions of adjustment capital, reinsurance and investment strategy for t, where $t = 1,2,\ldots T$.

The Optimization Model with Constraint of Solvency II

We use VaR ($1 - \alpha = 99.5\%$) of the net asset as the SCR defined in Solvency II. Given confidence level $\alpha \in (0,1)$, the VaR of the net assets at the confidence level $1 - \alpha$ is given by the smallest number l such that the probability of the loss X exceeding l is not larger than α. The SCR at time t is:

$$SCR_t = VaR_\alpha(\Delta X_t) = -\inf\{\Delta X_t \in \Re : P(\Delta X_t > l) \leq \alpha\} = -\inf\{l \in \Re : F_{\Delta X_t}(l) \geq \alpha\} \tag{4}$$

5. The factors affecting retention rate of reinsurance include the investment rate of return and risk, the underwriting risk, and the price and quantity demanded of insurance products.

6. For multi-period optimization models, it is important to consider the adjustment cost because the adjustment cost will affect the interval of adjusting the capital to the target value. (Please see Leary and Roberts, 2005; Flannery and Rangan, 2006; and Strebulaev, 2007). We thank an anonymous reviewer for this comment.

where X (t) satisfies stochastic differential equation either (3) and $\Delta X_t = X(t) - X(t-1)$.

Similar to the discussion above, we establish the objective function of minimizing the total frictional cost with the constraint that Pr $(\Delta X_t \leq -SCR_t) = \alpha$, that is,

$$\min \quad FC_t = \sum_{j=1}^{t} P_j(1-a)\eta + K(t)c_c - c_f E\big(X(t)/X(t)<0\big) + C_a \tag{5}$$

subject to:

$$\mathbf{Pr}\big(\Delta X_t \leq -SCR_t\big) = \alpha, t = 1, 2, ..., T\ .$$

With simulation, we can reach the optimal solutions of the reinsurance retention ratio and the proportion of risky assets invested by the insurer and SCR defined in Solvency II.

The Optimization Model with Constraint of the SST

The SST proposes the concept of target capital, which equals the one-year risk capital defined as the expected shortfall of the change in risk-bearing capital during a one-year period. The risk-bearing capital is defined as the difference between the market-consistent value of the assets and the best estimate of the liabilities.

Based on the definitions above, we establish the formula for calculating the target capital (TC) at time t:

$$TC_t = ES_\alpha = \frac{1}{\alpha}\int_0^\alpha VaR_\gamma(\Delta X_t)d\gamma \tag{6}$$

where X_t satisfies stochastic differential equation (3), $\Delta X_t = X_t - X_{t-1}$ and ES_α is the expected shortfall with confidence level of α. We use γ as the variable of the integral.

We establish the objective function of minimizing the total frictional cost with the following constraint:

$$TC_t = ES_\alpha = \frac{1}{\alpha}\int_0^\alpha VaR_\gamma(\Delta X_t)d\gamma, t = 1, 2, \ldots, T \text{ .}$$

That is,

$$\min \; FC_t = \sum_{j=1}^{t} P_j(1-a)\eta + K(t)c_c - c_f E\left(X(t)/X(t)<0\right) + C_a \tag{7}$$

subject to:

$$K(t) \geq TC_t \text{ .}$$

Numerical Analysis Based on an Example

In this section, we present an example to illustrate the optimal results and make comparisons among models based on different criteria discussed above.[7] For the ratio of capital cost, we use the results from Kielholz (2000), which is the average ratio of capital cost for nonlife insurance companies in 1998. (See Table 1 of Kielholz (2000).) For the bankruptcy cost, we use the results of Lewis (2009), who employs sample data from 1985–2005 to estimate the ratio of total bankruptcy cost to firm value. We use Standard & Poor's (S&P) 500 stock market index data and rates of return for Treasury bills during the period 1983–2007 to estimate the drift and the volatility of the rate of return on risky assets (stocks) and the risk-free interest rate. For other data (described below), we rely on results from Chandra and Sherris (2006). We focus on calculating the optimal adjustment capital required in the first year, which can be seen as the single-period model similar to the single-period model in Chandra and Sherris (2006). That is, we assume that there are no costs associated with adjusting to the optimal level of capital and consider only the frictional cost of capital and the cost of financial distress. Table 2 lists the input data for the analysis.

7. The information used in the numerical analysis here is drawn from prior studies and on estimates made by the authors; the results may vary with different underlying asssumptions.

Table 2:
The Input Data for the Example

Adjustment costs	0
Ratio of frictional costs of capital c_c	13.18%
Initial capital $X(0)$	1
Ratio of total bankruptcy cost to firm value c_f	16.3%
Expected claim payment $E(P_0)$	20
Premium income scenario 1 $P_1(1)$	10
Premium income scenario 2 $P_1(2)$	20
Premium income scenario 3 $P_1(3)$	25
Volatility of claim loss σ_2	4
Volatility of the return rate of risky assets(stocks) σ_1	0.1557
Drift of the return rate of risky assets μ	0.1381
Risk-free interest rate r	0.0512

Reinsurance contracts are privately negotiated, and the contract conditions may vary widely. The factors affecting the rate of reinsurance cost is very complicated. Because exact rates of reinsurance cost η are unknown to us, we calculate optimal solutions based on a range of values for each η. We also give the range of the rate of reinsurance cost, which is not economical to the insurer or does not satisfy the requirement of insolvency regulation with lowest total frictional cost and vice versa. The optimal solutions, *, are listed in Table 3, Table 4 and Table 5. From Table 3, we find that the rate of reinsurance cost, η, and the premium income, P_1, are important factors that affect the optimal results. When the rate of reinsurance cost is small or the premium income is small, the optimal retention rate a^* is small or equal to zero, which means the reinsurance partially or completely replaces capital to transfer claim risk.

From Table 3, we also find when the level of premuim income is low ($P_1 = 10$ in our case), the adjustment capital required is positive and increases with the increase of the rate of reinsurance cost. This means it is necessary for the insurer to increase capital so as to offset underwriting risk. Higher reinsurance cost will decrease the net income of the insurer and also prompt the insurer to increase capital held so as to reduce the insolvency risk caused by higher reinsurance cost.

Table 3:
Optimal Solutions Obtained by Optimization Model Without Constraint

	$K(1)^*$			π^*			a^*		
	$P_1=10$	$P_1=20$	$P_1=25$	$P_1=10$	$P_1=20$	$P_1=25$	$P_1=10$	$P_1=20$	$P_1=25$
$\eta=0.05$	0	-1.1	-6.8	0	0	0	0.00	1	1
$\eta=0.10$	0.5	-1.1	-6.8	0.40	0	0	0.05	1	1
$\eta=0.15$	2.0	-1.1	-6.8	0.30	0	0	0.15	1	1
$\eta=0.20$	9.7	-1.1	-6.8	0.15	0	0	1	1	1
$\eta=0.25$	10.3	-1.1	-6.8	0.10	0	0	1	1	1
$\eta=0.30$	10.3	-1.1	-6.8	0.10	0	0	1	1	1

From Table 3, we also find that when the level of premuim income is low ($P_1 = 10$ in our case) and when the rate of reinsurance cost is low ($\eta = 0.05$ in our case), the optimal retention rate is zero, and there is no risky investment. This means that it is optimal for the insurer to transfer all underwriting risk to the reinsurance due to the low reinsurance cost. However, when the rate of reinsurance cost increases, the investment strategy changes from more aggressive to more conservative, the retention portion is increasing, and the capital held is increasing. When $\eta \geq 0.2$, the optimal retention rate is 1, the optimal capital is great, and underwriting risk is reduced by increasing capital instead of buying reinsurance due to higher reinsurance cost.

Finally, from Table 3, we found that when the level of premium income is higher ($P_2 = 20$, and $P_3 = 25$ in our cases), the optimal capital is negative, which means that it is optimal for the insurer to reduce capital held so as to decrease capital cost. Also, higher premium income can be used to offset insolvency risk. In addition, the optimal retention rate is equal to 1 whether the rate of reinsurance cost is lower or higher. And all surplus is invested in risk-free assets. This suggests that the premium income and the return of risk-free assets are great enough to offset insolvency risk.

Table 4:
Optimal Solutions Obtained by Optimization Model Based on Solvency II

	$K(1)^*$			π^*			a^*		
	$P_1=10$	$P_1=20$	$P_1=25$	$P_1=10$	$P_1=20$	$P_1=25$	$P_1=10$	$P_1=20$	$P_1=25$
$\eta=0.05$	0	2.1	4.9	0	0.25	0.50	0	0.20	1
$\eta=0.10$	0	9.6	4.9	0	0.40	0.50	0	0.95	1
$\eta=0.15$	0.6	10.3	4.9	0	0.40	0.50	0	1	1
$\eta=0.20$	2.1	10.3	4.9	0.45	0.40	0.50	0.05	1	1
$\eta=0.25$	4.7	10.3	4.9	0.45	0.40	0.50	0.15	1	1
$\eta=0.30$	11.0	10.3	4.9	0.40	0.40	0.50	0.45	1	1

Note: P_1 – premium income; η– the rate of reinsurance cost; $K(1)^*$ – the optimal adjustment capital required; π^* – the optimal portion of risky assets; a^* – the optimal portion of retention.

Table 5:
Optimal Solutions Obtained by the Optimization Model Based on the Swiss Solvency Test

*	$K(1)^*$			π^*			a^*		
	$P_1=10$	$P_1=20$	$P_1=25$	$P_1=10$	$P_1=20$	$P_1=25$	$P_1=10$	$P_1=20$	$P_1=25$
$\eta=0.05$	0	3.1	5.7	0	0.35	0.30	0	0.35	0.95
$\eta=0.10$	0	10.2	5.9	0	0.35	0.55	0	0.90	1
$\eta=0.15$	0.6	11.3	5.9	0	0.50	0.55	0	1	1
$\eta=0.20$	5.9	11.3	5.9	0.35	0.50	0.55	0.25	1	1
$\eta=0.25$	6.3	11.3	5.9	0.40	0.50	0.55	0.30	1	1
$\eta=0.30$	11.4	11.3	5.9	0.45	0.50	0.55	0.45	1	1

Note: P_1 – premium income; η– the rate of reinsurance cost; $K(1)^*$ – the optimal adjustment capital required; π^* – the optimal portion of risky assets; a^* – the optimal portion of retention.

From Table 4 and Table 5, we find that the optimal adjustment capital required is greater with the constraints of Solvency II and the SST than without these constraints of insolvency. The tables illustrate that Solvency II and the SST tend to be more prudent in terms of determining optimal capital level.

From Table 4 and Table 5, we also find that the investment strategies are more aggressive when optimal adjustment capital required is greater or the premium income is greater. The higher capital and higher premium income can help the insurer to hedge the investment risk so as to allow for more aggressive investment strategies. All of these findings provide insight on the decision of optimal capital, investment and reinsurance strategies.

Next, we perform a sensitivity analysis of the optimal adjustment capital required, investment and reinsurance strategies to the change in several critical parameters, including the ratio of the frictional cost of capital, c_c; the rate of reinsurance cost, η; the drift of the rate of return on risky assets, μ; the volatility of the return rate of the risky assets invested, σ; the volatility of claim loss rate, σ_1; and the ratio of total bankruptcy cost to firm value, c_f.

The Effect of Changes in c_c, η

Table 6 and Table 7 display the optimal values of the proportions of reinsurance retention and the risky assets invested and the capital adjusted by the insurer for different values of the ratio of capital cost c_c and the rate of reinsurance cost η, when we use the criteria of minimizing the total control cost. The confidence level is set equal to 0.5% for Solvency II and 1% for the SST. We present the results for the second scenario of premium income. (The analyses of the other two scenarios of premium income are similar.)

The results in Table 6 show that the optimal investment strategy, π^*, tends to be more aggressive when the ratio of capital cost, c_c, is small and the optimal adjustment capital required, $K(1)^*$, is large with the optimization model without constraint of insolvency. (The optimal portion of risky assets (π^*) equals 0.55 when the ratio of capital cost, c_c, is 0.02, and the optimal adjustment capital required, $K(1)^*$, is 13.5.) However, the investment strategies tend to be more conservative with increases in the ratio of capital cost and with decreases in the optimal adjustment capital required. The portion of risky assets becomes zero when the optimal adjustment capital required is very small or negative. Because lower capital level means higher insolvency risk, the insurer should select a more conservative investment strategy in order to reduce investment risk and further to reduce insolvency risk. However, based on the estimation from the second and third model (with constraint of insolvency), the optimal capital levels are much higher than those estimated by the first model (without constraint of insolvency).

Table 6:
Optimal Capital Level, Reinsurance Retention Ratio and Risky Assets Invested by the Insurer When the Ratio of Reinsurance Cost is Greater than 0.15

Optimal model without constraint of insolvency $\eta \geq 0.15$								
r_c	0.02	0.04	0.06	0.08	0.10	0.12	0.14	0.16
a^*	1	1	1	1	1	1	1	1
π^*	0.55	0.40	0.15	0	0	0	0	0
$K(1)^*$	13.5	13.00	5.2	1.1	0	-1.1	-1.1	-1.2
$E(X(1))^*$	18.547	16.720	6.5276	2.0716	0.088	0.0416	0.0456	0.0262
$\Pr^*(X(t)<0, 0 \leq t \leq 1)$	0	0	0.0768	0.3053	0.4976	0.4981	0.4984	0.4942
FC_1^*	0.2700	0.5200	0.5918	0.5233	0.4367	0.4176	0.3875	0.3558
Optimal model based on Solvency II $\eta \geq 0.15, \alpha = 0.005$								
r_c	0.02	0.04	0.06	0.08	0.10	0.12	0.14	0.16
a^*	1	1	1	1	1	1	1	1
π^*	0.55	0.40	0.35	0.35	0.45	0.45	0.55	0.50
$K(1)^*$	13.5	13.00	9.7	9.9	10.1	10.2	10.3	10.4
$E(X(1))^*$	18.547	16.720	11.433	11.777	12.496	12.198	12.018	12.092
FC_1^*	0.2700	0.5200	0.8117	1.0178	1.2275	1.4568	1.6944	1.9500
Optimal model based on Swiss Solvency Test $\eta \geq 0.15, \alpha \leq 0.01$								
r_c	0.02	0.04	0.06	0.08	0.10	0.12	0.14	0.16
a^*	1	1	1	1	1	1	1	1
π^*	0.55	0.40	0.45	0.40	0.45	0.45	0.55	0.50
$K(1)^*$	13.5	13.00	11.0	11.2	11.3	11.3	11.4	11.5
$E(X(1))^*$	18.547	16.720	14.477	14.40	14.328	13.576	13.188	13.098
FC_1^*	0.2700	0.5200	0.7889	0.9912	1.2347	1.4815	1.7271	1.8948

Table 6 note: α – the confidence level; η – the rate of reinsurance cost; $K(1)^*$ – the optimal adjustment capital required; π^* – the optimal portion of risky assets; a^* – the optimal portion of retention; $FC_1{}^*$ – the optimal total friction cost; $E(X(1))^*$ – the optimal expected surplus at time 1; $\Pr^*(X(t)<0, 0\le t\le 1)$ – the optimal insolvency probability; μ – the drift of return rate of risky assets; σ_1 – the volatility of return rate of risky assets; r – the risk-free interest rate; σ_2 – the volatility of claim loss; c_c – the ratio of frictional capital cost; c_f – the ratio of the total bankruptcy cost to firm value.

However, it is slightly decreasing with the increase of the rate of capital cost, and the optimal investment strategies are more aggresive than those determined by the first model for most cases—except for two cases in which the rate of capital cost is very low (r_c = 0.02, 0.04 in our cases). Given the constraint of low probability of insolvency, the insurer must hold more capital to reduce insolvency risk, even though the capital is expensive. The higher capital level allows the insurer to follow a more aggressive investment strategy so as to increase investment return and decrease total frictional cost.

In addition, the total cost FC_1 (frictional cost of capital and the cost of bankrupcty) is larger for Solvency II and the SST than that for the optimization model without constraint of insolvency. The exceptions are the cases in which the ratio of capital cost $c_c \le 0.04$ and the optimal adjustment capital required by Solvency II and the SST, $K(1)^*$, is higher than what was determined by the optimal model without constraint of insolvency—except for the cases where the ratio of capital cost $c_c \le 0.04$. That is, in most cases, Solvency II and the SST are more prudent in determining optimal adjustment capital required than by only minimizing the total frictional cost.

The results of Table 6 also show that the optimal insolvency probabilities, $\Pr^*$ $(X(t) < 0,\ 0 \le t \le 1)$, determined by the optimization model without constraint of insolvency, become larger when the ratio of capital costs increases. Therefore, regulation based on Solvency II or the SST appear to be more binding in this situation. Finally, the results show that the optimal adjustment capital required by the SST is slightly greater than that from the Solvency II results.

The results in Table 7 indicate that if the frictional cost of capital is very high or the reinsurance cost is very low, reinsurance can partially replace the capital need and effectively serve as an instrument of capital management. (See the values in the last four columns corresponding optimal retention ratios a^*, which are small.) Table 7 indicates that most of the premium is ceded to the reinsurer so as to transfer claim risk.

Table 7:
Optimal Capital Level, Reinsurance Retention Ratio and Risky Assets Invested by the Insurer When the Ratio of Reinsurance Cost is 0.05

Optimal model without constraint of insolvency $\eta = 0.05$								
r_c	0.02	0.04	0.06	0.08	0.10	0.12	0.14	0.16
a^*	1	1	1	1	1	1	1	1
π^*	0.55	0.55	0.15	0	0	0	0	0
$K(1)^*$	13.5	13.0	5.2	1.1	0	-1.1	-1.1	-1.2
$E(X(1))^*$	18.547	16.720	6.5276	2.0716	0.088	0.0416	0.0456	0.0262
$\Pr^*(X(t)<0, 0 \le t \le 1)$	0	0	0.0768	0.3053	0.4976	0.4981	0.4984	0.4942
FC_1^*	0.2700	0.5200	0.5918	0.5233	0.4367	0.4176	0.3875	0.3558
Optimal model based on Solvency II $\eta = 0.05, \alpha = 0.005$								
r_c	0.02	0.04	0.06	0.08	0.10	0.12	0.14	0.16
a^*	1	1	1	1	0.35	0.20	0.15	0.10
π^*	0.55	0.40	0.30	0.30	0.40	0.40	0.35	0.30
$K(1)^*$	13.5	13.00	9.7	9.9	3.6	2.1	1.6	1.1
$E(X(1))^*$	18.547	16.720	11.433	11.777	3.7861	2.9447	1.7908	1.2068
FC_1^*	0.2700	0.5200	0.8117	1.0178	1.0810	1.1088	1.1046	1.0991
Optimal model based on the Swiss Solvency Test $\eta = 0.05, \alpha \le 0.01$								
r_c	0.02	0.04	0.06	0.08	0.10	0.12	0.14	0.16
a^*	1	1	1	1	0.45	0.40	0.35	0.30
π^*	0.55	0.40	0.45	0.40	0.35	0.35	0.40	0.35
$K(1)^*$	13.5	13.00	11.0	11.2	4.9	4.4	4.0	3.7
$E(X(1))^*$	18.547	16.720	14.477	14.40	5.7849	5.6050	5.4353	5.3248
FC_1	0.2700	0.5200	0.7889	0.9912	1.0733	1.1622	1.1749	1.2244

Table 7 note: α – the confidence level; η – the rate of reinsurance cost; $K(1)^*$ – the optimal adjustment capital required; π^* – the optimal portion of risky assets; a^* – the optimal portion of retention; FC_1^* – the optimal total friction cost; $E(X(1))^*$ – the optimal expected surplus at time 1; $\Pr^*(X(t)<0, 0\leq t\leq 1)$ – the optimal insolvency probability; μ – the drift of return rate of risky assets; σ_1 – the volatility of return rate of risky assets; r – the risk-free interest rate; σ_2 – the volatility of claim loss; c_c – the ratio of frictional capital cost; c_f – the ratio of the total bankruptcy cost to firm value.

Table 7 also indicates that the investment strategies determined by the second model and third model are more aggressive than those determined by the first model for most cases—except for two cases that the rate of capital cost is very low (r_c = 0.02, 0.04 in our cases). Higher capital level or a higher portion of reinsurance can decrease probability of insolvency and underwiring risk so as to allow the insurer to take more aggressive investment strategy, obtain more investment return and decrease total frictional cost.

Figure 1 depicts the relationship between the rate of frictional cost of capital, the probability of insolvency based on optimal model without constraint of insolvency, and the confidence level of Solvency II and the SST from the numerical results of the example.

Figure 1:
The Relationship Between the Rate of Frictional Cost of Capital, the Probability of Insolvency Based on a Model Without Constraint of Insolvency, and the Confidence Levels for Solvency II and the Swiss Solvency Test ($\eta \geq 0.15$)

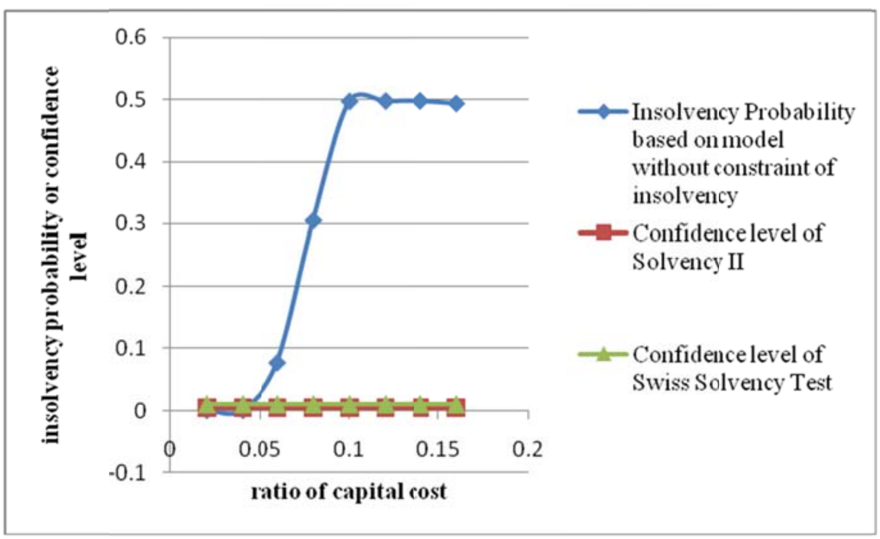
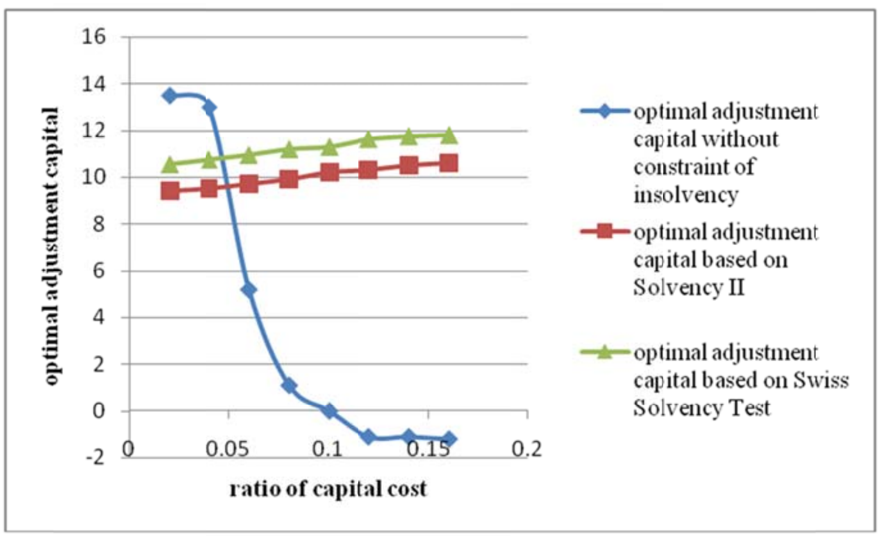


Figure 1 shows that the probability of insolvency based on the optimization model without constraint of insolvency is much greater than with VaR constraint based on Solvency II and with tail value at risk (TVaR) constraint based on the SST in most cases—except for the cases where the ratio of capital cost $c_c \leq 0.04$. The results also indicate that setting the regulatory capital level based on either Solvency II or the SST leads to greater prudence toward the risk of insolvency than determining capital by only minimizing the total frictional cost—except for the cases where the ratio of capital cost is very small.

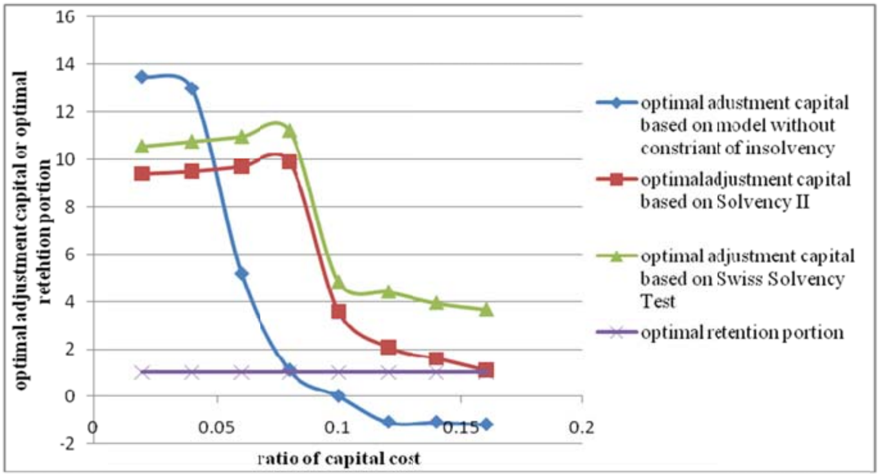
Figure 2 describes the relationship between the optimal adjustment capital determined by the optimization model without constraint of insolvency, based on Solvency II and the SST and the rate of frictional cost of capital when the cost of reinsurance for each exposure $\eta \geq 0.15$[8] and other data as described above in the example. The results in Figure 2 indicate that the optimal adjustment capital determined by the optimization model without constraint of insolvency decreases as the rate of frictional cost of capital, c_c, increases, since larger frictional cost of capital will make the total control cost increase, and it is necessary to decrease capital in order to keep the total control cost minimized.

Figure 2:
The Relationship Between Optimal Adjustment Capital and the Ratio of Capital Cost of Three Models (η ≥ 0.15)



8. The reason that we choose $\eta \geq 0.15$ is that the optimal retention portion of reinsurance is equal to 1 when $\eta \geq 0.15$. In order to make analysis and comparison conveniently, we first discuss the cases without reinsurance. In Figure 3 and Figure 5 (also see Table 7), we will set a smaller value of η and discuss the cases with reinsurance.

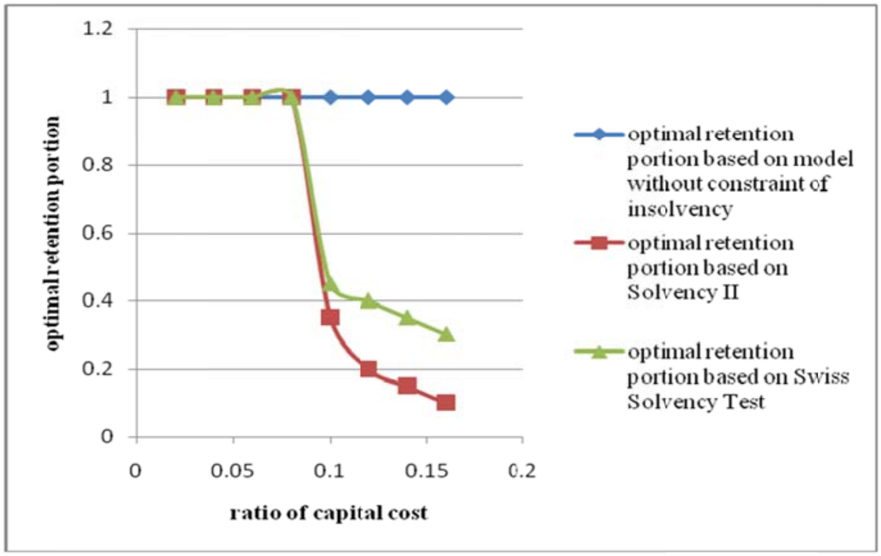
Figure 3:
The Change Patterns in Optimal Adjustment Capital of Three Models When the Ratio of the Frictional Cost of Capital Changes (η = 0.05)



The results in Figure 2 also show that the optimal adjustment capital required by Solvency II (SCR) increases with the increase of c_c since surplus decreases and the probability of default increases when c_c increases, and it is necessary to increase capital in order to keep the default probability equal to 0.005 constantly. Finally, Figure 2 shows that the optimal adjustment capital determined based on the SST is slightly greater than that based on Solvency II, and the proportion of retention of reinsurance is equal to 1 for all three models.

Figure 3 and Figure 4 illustrate the changes in optimal adjustment capital and optimal proportion of retention of reinsurance for three models respectively, when the reinsurance cost of each exposure is 0.05. From Figure 3 and Figure 4, we find that reinsurance will partially replace capital needs and may serve as an effective instrument of capital management when the cost of reinsurance is relatively low and the frictional cost of capital is relatively large for these three models. Figure 3 and Figure 4 also show that the optimal strategy of capital and reinsurance is quite different based on Solvency II and the SST when the frictional cost of capital is high and the reinsurance cost is low.

Figure 4:
The Change Patterns in Optimal Proportion of Retention of Three Models When the Ratio of the Frictional Cost of Capital Changes (η = 0.05)



The Effect of Changes in μ, σ_1, r, σ_2 and c_f

Table 8, Table 9 and Table 10 list the optimal adjustment capital required, the proportions of reinsurance retention and the risky assets invested by the insurer when the parameters of μ, σ_1, r, σ_2 and c_f change based on three models, respectively. The data in Table 8 show that the optimal adjustment capital required, $K(1)^*$, remains constant when parameters change. The optimal portion of risky assets, π^*, is zero when parameters change, and the main reason is that the optimal adjustment capital required is negative. Because there is no risky investment, all optimal solutions remain unchanged when the return rate and volatility of risky assets, μ and σ_1, change. The optimal total frictional cost, FC_1^*, increases, but optimal probability of insolvency, $\Pr^*(X(t) < 0, 0 \le t \le 1)$, is almost unchanged when the risk-free interest rate, r, and the ratio of bankruptcy cost to the firm value, c_f, increase.

It is important to notice that the optimal values of $\Pr^*(X(t) < 0, 0 \le t \le 1)$ and FC_1^* increase, but the optimal capital level remains unchanged as the volatility of claim loss, σ_2, increases, and optimal retention portions are all equal to 1 no matter what value of claim loss, σ_2, takes. These results illustrate that high reinsurance cost ($\eta = 0.15$ in our cases) will lead the insurer to retain all premium income even in the situations of greater claim risk. The optimal economic strategy for the insurer is to keep capital unchanged, but this approach will increase the insurer's probability of insolvency when the volatility of claim loss, σ_2, increases. This is the limitation of model one. Minimizing total frictional cost from the insurer's perspective may conflict with the objective of regulation. The data in Table 8 also show that the optimal investment strategy is that all surplus is invested in risk-free assets. The return of risk-free assets can help to offset part of claim risk and help to reduce total frictional cost.

Table 9 indicates that with the constraint of Solvency II, the optimal capital, $K(1)^*$, slightly decreases with the increase of the drift of return rate of risky assets, μ, and slightly increases with the increase of volatility of return rate of risky asset, σ_1, and risk-free interest rate, r. And the optimal adjustment capital is much greater than determined earlier based on the optimization model without constraint of insolvency. From Table 9, we also observe that the optimal solution of $K(1)^*$ greatly increases as the volatility of claim loss, σ_2, increases. The increase of claim risk will lead to increased capital in order to keep the insolvency probability within the criterion of Insolvency II. But the optimal retention portion are all equal to 1 for all values of σ_2, meaning that it is optimal for the insurer to raise capital instead of purchasing reinsurance in order to reduce underwriting risk due to higher rate of reinsurance cost. Therefore, based on the analysis above, we illustrate based on various assumptions and with regard to varying regulatory standards that capital and reinsurance can be substitutes. On the one hand, when reinsurance cost is rather low but capital cost is rather high, reinsurance can partially or fully substitute for capital, as shown in Table 7. On the other hand, capital can partially or fully substitute for reinsurance when capital cost is rather low but the reinsurance cost is rather high, as in Table 6. Finally, we find from Table 9 that optimal adjustment capital remains fairly constant when the ratio of total bankruptcy cost to firm value, c_f, increases, but the total frictional cost, FC_1^*, increases with the increase of c_f.

From Table 10, we find that the change patterns are similar to those in Table 9 when the parameters change. However, the optimal adjustment capital, $K(1)^*$, and the optimal total frictional cost, FC_1^*, are slightly greater than those in Table 9. That is, the criteria of the SST are slightly more prudent in determining the optimal adjustment capital.

It should be noticed that Table 8, Table 9 and Table 10 indicate that the optimal total frictional cost (and other optimal solutions) are very sensitive to the change of the volatility of claim loss, σ_2, but the optimal frictional cost (and other optimal solutions) changes very slightly as other paramters change, given the condition that the rate of reinsurance cost $\eta = 0.15$. So, σ_2 is the most important parameter affecting total frictional cost. In the following, we will further discuss the sensitivity of the optimal solutions as σ_2 increases when η takes smaller values ($\eta = 0.10$) for three models, respectively.

Table 8:
Optimal Adjustment Capital Required, Reinsurance Retention Ratio and Risky Assets Invested by the Insurer with Varying Parameters of μ, σ_1, r, σ_2 and c_f Based on the Optimization Model Without Insolvency Constraint

$\eta = 0.15, r_c = 0.1318$								
μ	**0.08**	**0.09**	**0.10**	**0.11**	**0.12**	**0.13**	**0.14**	**0.15**
a^*	1	1	1	1	1	1	1	1
π^*	0	0	0	0	0	0	0	0
$K(1)^*$	-1.1	-1.1	-1.1	-1.1	-1.1	-1.1	-1.1	-1.1
$E(X_1^*)$	0.0350	0.0350	0.0350	0.0350	0.0350	0.0350	0.0350	0.0350
$\Pr^*(X(t) < 0, 0 \le t \le 1)$	0.4989	0.4989	0.4989	0.4989	0.4989	0.4989	0.4989	0.4989
FC_1^*	0.3979	0.3979	0.3979	0.3979	0.3979	0.3979	0.3979	0.3979
σ_1	**0.10**	**0.11**	**0.12**	**0.13**	**0.14**	**0.15**	**0.16**	**0.17**
a^*	1	1	1	1	1	1	1	1
π^*	0	0	0	0	0	0	0	0
$K(1)^*$	-1.1	-1.1	-1.1	-1.1	-1.1	-1.1	-1.1	-1.1
$E(X_1^*)$	0.0350	0.0350	0.0350	0.0350	0.0350	0.0350	0.0350	0.0350
$\Pr^*(X(t) < 0, 0 \le t \le 1)$	0.4989	0.4989	0.4989	0.4989	0.4989	0.4989	0.4989	0.4989
FC_1^*	0.3979	0.3979	0.3979	0.3979	0.3979	0.3979	0.3979	0.3979
r	**0.01**	**0.02**	**0.03**	**0.04**	**0.05**	**0.06**	**0.07**	**0.08**
a^*	1	1	1	1	1	1	1	1
π^*	0	0	0	0	0	0	0	0
$K(1)^*$	-1.1	-1.1	-1.1	-1.1	-1.1	-1.1	-1.1	-1.1
$E(X_1^*)$	0.0344	0.0299	0.0327	0.0409	0.0273	0.0584	0.0374	0.014
$\Pr^*(X(t) < 0, 0 \le t \le 1)$	0.4963	0.4963	0.4965	0.4964	0.4965	0.4962	0.4967	0.4965
FC_1^*	0.3780	0.3811	0.3892	0.3925	0.3987	0.4028	0.4085	0.4177

Table 8 ***(Continued)***

σ_2	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**
a^*	1	1	1	1	1	1	1	1
π^*	0	0	0	0	0	0	0	0
$K(1)^*$	-1.1	-1.1	-1.1	-1.1	-1.1	-1.1	-1.1	-1.1
$E(X_1^*)$	0.0392	0.0342	0.0326	0.0339	0.0234	0.0250	0.0286	0.0309
$\Pr^*(X(t)<0, 0 \le t \le 1)$	0.4850	0.4941	0.4974	0.4982	0.4986	0.4989	0.4992	0.5004
FC_1^*	-0.0113	0.1261	0.2613	0.3989	0.5357	0.6699	0.8068	0.9461
c_f	**0.16**	**0.17**	**0.18**	**0.19**	**0.20**	**0.21**	**0.22**	**0.23**
a^*	1	1	1	1	1	1	1	1
π^*	0	0	0	0	0	0	0	0
$K(1)^*$	-1.1	-1.1	-1.1	-1.1	-1.1	-1.1	-1.1	-1.1
$E(X_1^*)$	0.0407	0.0418	0.453	0.0376	0.0389	0.0403	0.0440	0.0392
$\Pr^*(X(t)<0, 0 \le t \le 1)$	0.4967	0.4967	0.4966	0.4968	0.4964	0.4968	0.4970	0.4969
FC_1^*	0.3859	0.4225	0.4512	0.4909	0.5267	0.5521	0.5913	0.6203

Note: α – the confidence level; η – the rate of reinsurance cost; $K(1)^*$ – the optimal adjustment capital required; π^* – the optimal portion of risky assets; a^* – the optimal portion of retention; FC_1^* – the optimal total friction cost; $E(X(1))^*$ – the optimal expected surplus at time 1; $\Pr^*(X(t)<0, 0 \le t \le 1)$ – the optimal insolvency probability; μ – the drift of return rate of risky assets; σ_1 – the volatility of return rate of risky assets; r – the risk-free interest rate; σ_2 – the volatility of claim loss; c_c – the ratio of frictional capital cost; c_f – the ratio of the total bankruptcy cost to firm value.

Table 9:
Optimal Adjustment Capital Required, Reinsurance Retention Ratio and Risky Assets Invested by the Insurer with Varying Parameters of μ, σ_1, r, σ_2 and c_f Based on the Optimal Model with Solvency II

$\eta = 0.15, r_c = 0.1318, \alpha = 0.005$								
μ	**0.08**	**0.09**	**0.10**	**0.11**	**0.12**	**0.13**	**0.14**	**0.15**
a^*	1	1	1	1	1	1	1	1
π^*	0.30	0.35	0.45	0.45	0.35	0.45	0.30	0.35
$K(1)^*$	10.6	10.5	10.4	10.3	10.3	10.3	10.2	10.1
$E(X_1^*)_*$	11.873	11.608	12.093	12.421	11.319	12.414	11.620	11.769
FC_1^*	1.5706	1.5585	1.5529	1.5542	1.5670	1.5740	1.5490	1.5609
σ_1	**0.10**	**0.11**	**0.12**	**0.13**	**0.14**	**0.15**	**0.16**	**0.17**
a^*	1	1	1	1	1	1	1	1
π^*	0.60	0.60	0.60	0.55	0.55	0.50	0.50	0.50
$K(1)^*$	10.0	10.2	10.3	10.3	10.3	10.3	10.5	10.6
$E(X_1^*)$	10.869	11.099	11.383	11.217	11.028	11.114	11.367	11.38
FC_1^*	1.5413	1.5389	1.5422	1.5664	1.5400	1.5807	1.5574	1.5651
r	**0.01**	**0.02**	**0.03**	**0.04**	**0.05**	**0.06**	**0.07**	**0.08**
a^*	1	1	1	1	1	1	1	1
π^*	0.55	0.55	0.45	0.45	0.50	0.55	0.55	0.55
$K(1)^*$	10.0	10.1	10.2	10.2	10.3	10.4	10.5	10.5
$E(X_1^*)$	12.308	12.292	12.034	12.354	12.467	12.515	12.799	12.9287
FC_1^*	1.5754	1.5443	1.5540	1.5420	1.5493	1.5599	1.5454	1.5309

Table 9 ***(Continued)***

σ_2	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**
a^*	1	1	1	1	1	1	1	1
π^*	0.15	0.60	0.60	0.55	0.45	0.55	0.45	0.55
$K(1)^*$	1.8	4.6	7.0	10.3	13.0	15.1	18.2	21.4
$E(X_1^*)$	2.888	6.6015	9.3468	12.627	15.124	18.070	22.233	25.103
FC_1^*	0.2926	0.6552	1.1324	1.5604	1.9355	2.4306	2.7453	3.1909
c_f	**0.16**	**0.17**	**0.18**	**0.19**	**0.20**	**0.21**	**0.22**	**0.23**
a^*	1	1	1	1	1	1	1	1
π^*	0.45	0.50	0.40	0.60	0.40	0.40	0.65	0.55
$K(1)^*$	10.2	10.2	10.3	10.3	10.3	10.3	10.3	10.3
$E(X_1^*)$	12.403	12.321	12.154	12.415	12.019	12.104	12.872	12.576
FC_1^*	1.5508	1.5836	1.6121	1.6209	1.6310	1.6545	1.6715	1.6913

Note: α – the confidence level; η – the rate of reinsurance cost; $K(1)^*$ – the optimal adjustment capital required; π^* – the optimal portion of risky assets; a^* – the optimal portion of retention; FC_1^* – the optimal total friction cost; $E(X(1))^*$ – the optimal expected surplus at time 1; $\Pr^*(X(t)<0, 0\le t\le 1)$ – the optimal insolvency probability; μ – the drift of return rate of risky assets; σ_1 – the volatility of return rate of risky assets; r – the risk-free interest rate; σ_2 – the volatility of claim loss; c_c – the ratio of frictional capital cost; c_f – the ratio of the total bankruptcy cost to firm value.

Table 10:
Optimal Adjustment Capital Required, Reinsurance Retention Ratio and Risky Assets Invested by the Insurer with Varying Parameters of μ, σ_1, r, σ_2 and c_f Based on the Optimal Model with the Swiss Solvency Test

$\eta = 0.15, r_c = 0.1318, \alpha \le 0.01$								
μ	**0.08**	**0.09**	**0.10**	**0.11**	**0.12**	**0.13**	**0.14**	**0.15**
a^*	1	1	1	1	1	1	1	1
π^*	0.35	0.40	0.50	0.55	0.30	0.40	0.30	0.35
$K(1)^*$	11.6	11.6	11.5	11.5	11.4	11.3	11.3	11.2
$E(X_1^*)$	13.548	13.576	13.314	13.633	12.676	13.188	12.522	12.584
FC_1^*	1.6683	1.6418	1.6394	1.6157	1.6073	1.6065	1.5960	1.5979
σ_1	**0.10**	**0.11**	**0.12**	**0.13**	**0.14**	**0.15**	**0.16**	**0.17**
a^*	1	1	1	1	1	1	1	1
π^*	0.60	0.55	0.60	0.55	0.55	0.55	0.55	0.60
$K(1)^*$	10.9	11.1	11.1	11.2	11.2	11.2	11.2	11.3
$E(X_1^*)$	13.090	12.600	13.010	12.466	13.115	12.681	12.920	13.090
FC_1^*	1.5959	1.5914	1.6165	1.6268	1.6142	1.6262	1.6060	1.6116
r	**0.01**	**0.02**	**0.03**	**0.04**	**0.05**	**0.06**	**0.07**	**0.08**
a^*	1	1	1	1	1	1	1	1
π^*	0.50	0.50	0.40	0.40	0.35	0.30	0.45	0.50
$K(1)^*$	11.0	11.1	11.2	11.2	11.3	11.3	11.4	11.4
$E(X_1^*)$	12.567	12.899	12.629	12.854	12.819	13.108	13.251	13.777
FC_1^*	1.6228	1.6184	1.6263	1.6220	1.6291	1.6285	1.6290	1.6377

Table 10 ***(Continued)***

σ_2	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**
a^*	1	1	1	1	1	1	1	1
π^*	0.55	0.35	0.45	0.50	0.45	0.40	0.30	0.30
$K(1)^*$	2.1	5.1	8.2	11.2	14.1	16.9	20.2	23.6
$E(X_1^*)$	3.188	6.508	10.128	13.382	16.595	18.885	22.300	25.854
FC_1^*	0.3143	0.7284	1.2098	1.6157	2.0420	2.4905	2.9222	3.3940
c_f	**0.16**	**0.17**	**0.18**	**0.19**	**0.20**	**0.21**	**0.22**	**0.23**
a^*	1	1	1	1	1	1	1	1
π^*	0.35	0.50	0.30	0.60	0.50	0.45	0.45	0.30
$K(1)^*$	11.2	11.2	11.2	11.2	11.3	11.3	11.3	11.3
$E(X_1^*)$	12.956	13.323	12.639	13.953	13.624	13.248	13.136	13.566
FC_1^*	1.6245	1.6357	1.6463	1.6509	1.6760	1.6814	1.7033	1.7294

Note: α – the confidence level; η – the rate of reinsurance cost; $K(1)^*$ – the optimal adjustment capital required; π^* – the optimal portion of risky assets; a^* – the optimal portion of retention; FC_1^* – the optimal total friction cost; $E(X(1))^*$ – the optimal expected surplus at time 1; $\Pr^*(X(t)<0, 0\le t\le 1)$ – the optimal insolvency probability; μ – the drift of return rate of risky assets; σ_1 – the volatility of return rate of risky assets; r – the risk-free interest rate; σ_2 – the volatility of claim loss; c_c – the ratio of frictional capital cost; c_f – the ratio of the total bankruptcy cost to firm value.

Table 11 lists the results. Table 11 indicates that the optimal retention portion is still equal to 1 as σ_2 increases, although the rate of reinsurance cost decreases from 0.15 to 0.1 for model 1 (without constraint of insolvency). This means that reinsurance cannot help the insurer to decrease total frictional cost, so the insurer still retains all preimum income but invests all surplus in risk-free assets to increase income and reduce the risk of claim loss. Table 11 also indicates that for model 2 based on Insolvency II and model 3 based on the SST, optimal retention portion decreases with the increase of σ_2 and optimal capital level is much lower in that the rate of reinsurance cost is larger (η = 0.15) when $\sigma_2 \geq 4$. (See Table 9 and Table 10.) The insurer reduces the capital cost through reinsurance while satisfying the regulation requirement.

Table 11:
Optimal Adjustment Capital Required, Reinsurance Retention Ratio and Risky Assets Invested by the Insurer with Varying Parameter of σ_2 and Based on Three Optimization Models Respectively (Given η = 0.10)

Optimal model without constraint of insolvency $\eta = 0.10$								
σ_2	1	2	3	4	5	6	7	8
a^*	1	1	1	1	1	1	1	1
π^*	0	0	0	0	0	0	0	0
$K(1)^*$	-1.1	-1.1	-1.1	-1.1	-1.1	-1.1	-1.1	-1.1
$E(X_1^*)$	0.0392	0.0342	0.0326	0.0339	0.0234	0.0250	0.0286	0.0309
$\Pr^*(X(t)<0, 0 \le t \le 1)$	0.4850	0.4941	0.4974	0.4982	0.4986	0.4989	0.4992	0.5004
FC_1^*	-0.0113	0.1261	0.2613	0.3989	0.5357	0.6699	0.8068	0.9461
Optimal model based on Solvency II $\eta = 0.10, \alpha = 0.005$								
σ_2	1	2	3	4	5	6	7	8
a^*	1	1	1	0.74	0.60	0.10	0.06	0.04
π^*	0.15	0.60	0.60	0.50	0.40	0.80	0.60	0.40
$K(1)^*$	1.8	4.6	7.0	7.2	8.0	2.6	2.2	2.0
$E(X(1))^*$	2.888	6.6015	9.3468	8.5481	8.7147	2.0480	1.3640	1.0188
FC_1^*	0.2926	0.6552	1.1324	1.5522	2.0277	2.1880	2.1936	2.2019

Table 11 ***(Continued)***

Optimal model based on the Swiss Solvency Test $\eta = 0.10, \alpha \le 0.01$								
σ_2	1	2	3	4	5	6	7	8
a^*	1	1	1	0.85	0.78	0.40	0.06	0.06
π^*	0.55	0.35	0.45	0.38	0.46	0.45	0.48	0.30
$K(1)^*$	2.1	5.1	8.2	8.7	10	6.0	2.2	2.3
$E(X(1))^*$	3.188	6.508	10.128	10.097	11.5113	6.2598	1.4889	1.3866
FC_1	0.3143	0.7284	1.2098	1.5751	2.1281	2.1979	2.2076	2.2176

Note: α – the confidence level; η – the rate of reinsurance cost; $K(1)^*$ – the optimal adjustment capital required; π^* – the optimal portion of risky assets; a^* – the optimal portion of retention; FC_1^* – the optimal total friction cost; $E(X(1))^*$ – the optimal expected surplus at time 1; $\Pr^*(X(t)<0, 0\le t\le 1)$ – the optimal insolvency probability; μ – the drift of return rate of risky assets; σ_1 – the volatility of return rate of risky assets; r – the risk-free interest rate; σ_2 – the volatility of claim loss; c_c – the ratio of frictional capital cost; c_f – the ratio of the total bankruptcy cost to firm value.

Summary and Conclusion

In this paper, we established integrated models to determine the insurer's optimal adjustment capital required, reinsurance and investment strategy. We performed numerical analysis using simulation with a property-liability insurer as an example. The results show that reinsurance may be used as an effective instrument of capital management, particularly when the frictional cost of capital is high and/or the reinsurance cost is low. However, capital can partly or fully substitute for reinsurance when capital cost is low and reinsurance cost is high.

We find that the optimal investement strategy is for the insurer to accept more risk when the optimal adjustment capital required and/or the premium income are larger. Setting the regulatory capital level either by Solvency II or the SST leads to greater prudence than determining insurer capital level by minimizing total frictional cost in most cases, except when capital cost is very small. Future research may benefit from numerical analysis for life insurers.

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