Analyzing the Impact of Time Horizon, Volatility and Profit Margins on Solvency Capital: Proposing a New Model for the Global Regulation of the Insurance Industry

Thomas Mueller

Opinion Disclaimer

The views and opinions expressed here are those of the author and do not necessarily reflect the official policy or position of the NAIC or any other agency, organization, employer or company.

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The global Insurance Capital Standard (ICS) might include EU Solvency II concepts

The International Association of Insurance Supervisors (IAIS) will develop a global capital standard (ICS). It appears that ICS will incorporate the key points of European Solvency II-regulation:

1. The ICS is being developed with the aim of creating a common language for supervisory discussions.

2. Solvency II requirement: The solvency capital must cover risks with a given shortfall probability of 1/200 on a one-year time horizon. One year is very short compared to contractual terms in traditional life insurance of several decades.

Solvency II provides such a common language based on a market view for the balance sheet, publicly accessible with the SFCR reports for any insurance companies.
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   - **Solvency II provides such a common language based on a market view for the balance sheet, publicly accessible with the SFCR reports for any insurance companies.**
   - **With a short time horizon, the business is conducted too cautiously without aiming for a higher profit margin, which only reduces the risk in the long term.**

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**UK insurer AVIVA figures to show how Solvency II works:**
UK insurer AVIVA figures to show how Solvency II works:

### 2018 AVIVA Economic balance sheet

<table>
<thead>
<tr>
<th>Description</th>
<th>Billion £</th>
<th>Normalized to assets = 1</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Assets</strong></td>
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<td></td>
</tr>
<tr>
<td><strong>Liabilities</strong></td>
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<td></td>
</tr>
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<td></td>
</tr>
<tr>
<td><strong>SCR (Solvency Capital Requirement)</strong></td>
<td>15.3</td>
<td>Solvency Ratio: 27.6/15.3 = 180%</td>
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- **Description**:
  - Capital owned by stockholder
  - Capital at disposal to carry the risk
  - Minimal ratio is 100%, i.e. an eligible capital of at least 100%
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- Volatility σ = SCR%/2.58 = 3.9%/2.58
- Profit margin

A simple grid model for equity developments over multiple time steps
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0.8% + 1.5% = 2.3%

0.8% - 1.5% = -0.7%
A simple grid model for equity developments over multiple time steps

$2^{15} = 32,000$ scenarios (random walks)

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Expected value for the equity after 10 years
A simple grid model for equity developments over multiple time steps

2^{15} \approx 32,000 scenarios (random walks)

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0.8\% + 1.5\% = 2.3\%

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0\%

15\%

Time (years)

5

10

15

A simple grid model for equity developments over multiple time steps

2^{15} \approx 32,000 scenarios (random walks)

Expected value for the equity after 10 years

# random walks with given equity after 15 years for a depleted equity after 10 or 14 years

0.8\% + 1.5\% = 2.3\%

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0\%

15\%

Time (years)

5

10

15

10

15

1

4

10

10 + 9
A simple grid model for equity developments over multiple time steps

- $2^{15} = 32,000$ scenarios (random walks)

- **Expected value for the equity after 10 years**
  - $0.8\% + 1.5\% = 2.3\%$
  - $0.8\% - 1.5\% = -0.7\%$

- **Probability of ruin within 15 years**
  - $50(=34/16)/32,000 = 1.55\%$

- **Probability of shortfall after 15 years**
  - $16/32,000 = 0.5\%$
Concrete question for the business management of AVIVA

Assume that the obligations under AVIVA's life insurance contracts have an average term of 20 or 30 years.

In the long run: What will be the less risky business policy?

Current strategy

<p>| | |</p>
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Towards analytic models based on Brownian motion

Analytical models for stochastic development of equity with increasing deviations are Brownian motions.
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Probability density of a scenario leading to an equity \( x \), regardless the previous development.

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- A positive margin ($m>0$) → upwards motion
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\[
\begin{align*}
  p(x, t) &= \text{probability density of scenario leading to equity } x, \\
  p_+ &= \text{probability density after several decades}
\end{align*}
\]
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Probability of shortfall at time $t$ ⇔ Probability of a negative equity at time $t$ ⇔ area of the surface in green, if the whole surface in grey and green is normalized to 1.
Towards analytic models based on Brownian motion

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Probability density of a scenario leading to an equity $x$, regardless the previous development.

That needs to be corrected by introducing a function $p_-$ such that $p_+(0, t) - p_-(0, t) = 0$.

Correcting Brownian motion for the subtracted term $p_-(x, t)$

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Correcting Brownian motion for the subtracted term $p_-(x, t)$

$p_-(x, t)$:
Probability density of a scenario leading to an equity $x$ with crossing the zero line at least once.

$p_+(x, t) - p_-(x, t)$:
Probability density of a scenario leading to an equity $x$ without crossing the zero line before.
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After a few years
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Formula and geometric meaning for the probability of ruin
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Probability of shortfall at time $t$

$\Phi \left( -\frac{d + mt}{\sigma \sqrt{t}} \right)$
Formula and geometric meaning for the probability of ruin

Probability of shortfall at time $t$

$$\Phi\left(-\frac{d + mt}{\sigma \sqrt{t}}\right)$$

Probability of falling short before time $t$ and then recovering up to an equity $x > 0$ at time $t > 0$

$$e^{-\frac{2md}{\sigma^2}} \phi\left(-\frac{d + mt}{\sigma \sqrt{t}}\right)$$

Probability of going ruin up to time $t$

$$\psi(t)$$

Probability of shortfall at time $t$ $p_-$

Probability of falling short before time $t$ and then recovering up to an equity $x > 0$ at time $t > 0$ $p_+$

Probability of going ruin up to time $t$ $p_-$

$$p_+ - p_-(x, t)$$
Formula and geometric meaning for the probability of ruin

\[\Phi \left( \frac{d + mt}{\sigma \sqrt{t}} \right) + \frac{2mt}{\sigma^2} \phi \left( \frac{d + mt}{\sigma \sqrt{t}} \right)\]

Equity

\[p_-(x,t) = p_+(x,t)\]

Probability of shortfall at time \(t\) and then recovering up to an equity \(x > 0\) at time \(t > 0\)

The choice of business policy for

<table>
<thead>
<tr>
<th>Eligible capital or equity</th>
<th>(d)</th>
<th>7.0%</th>
</tr>
</thead>
<tbody>
<tr>
<td>Business Management according Solvency II</td>
<td>Volatility (\sigma)</td>
<td>1.5%</td>
</tr>
<tr>
<td>Profit margin (m)</td>
<td>0.8%</td>
<td></td>
</tr>
<tr>
<td>Probability Shortfall</td>
<td>1</td>
<td>0.00001%</td>
</tr>
<tr>
<td>Ruin = proposed model</td>
<td>0.00003%</td>
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<tr>
<td>Corresponding Solvency II ratio Shortfall</td>
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### The choice of business policy for Eligible capital or equity $d$ 7.0%

#### Business Management according Solvency II
- Volatility $\sigma$ 1.5%
- Profit margin $m$ 0.8%

#### Business Management according proposed model
- Volatility $\sigma$ 2.0%
- Profit margin $m$ 1.5%

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</tr>
<tr>
<td><strong>Shortfall</strong></td>
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| **Corresponding Solvency II ratio** | 180% | 109% | 118% | 129% | 141% |
| **Ruin = proposed model** | 175% | 89% | 85% | 84% | 84% |

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<td>0.0027%</td>
<td>0.50%</td>
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| **Corresponding Solvency II ratio** | 148% | 121% | 144% | 166% | 185% |
| **Ruin = proposed model** | 141% | 90% | 89% | 89% | 89% |

---

**Notes:**
- The choice of business policy depends on the probability of shortfall and ruin, as well as the corresponding Solvency II ratio.
- Higher values for Solvency II ratio indicate greater security against insolvency.
- The proposed model generally offers better protection against ruin compared to the Solvency II model.
The choice of business policy for

| Eligible capital or equity d | 7.0% |

Business Management according Solvency II
Volatility σ 1.5%
Profit margin m 0.8%

Business Management according proposed model
Volatility σ 2.0%
Profit margin m 1.5%

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<tr>
<td>20</td>
<td>0.037%</td>
<td>0.74%</td>
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<tr>
<td>30</td>
<td>0.010%</td>
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</tr>
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Solvency II forces insurers to invest in safe government bonds and not in more suitable corporate bonds with a safer long-term perspective. Or to reinsure a higher proportion of the business than would be appropriate in the long term.

Regulatory context of results – what does this mean for regulators?

Remarks:
Problems of the European Solvency II-Standard became hopefully more transparent.
The legislator decides; the regulators can influence the legislation and have considerable room for manoeuvre in implementing the laws.
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Options for actions – two ends of the spectrum:
Nation-state policy: Stick to the reasonable U.S. RBC standard for variable annuities
Acceptance of the future worldwide standard based on Solvency II; use workarounds to avoid the disadvantages.
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- Acceptance of the future worldwide standard based on Solvency II; use workarounds to avoid the disadvantages.

Many thanks for your attentions.
The Matching Adjustment as a workaround in Solvency II

UK life insurance market reveals major differences even in one country itself:

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<th>Products</th>
<th>Traditional participating policies</th>
<th>Future market</th>
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<td>Risk mitigation</td>
<td>Enormous discretion to reduce maturity value, see JIR article 38-04</td>
<td>Matching Adjustment exemption of Solvency II</td>
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In Solvency II, future payments are generally discounted at the risk-free interest rate. However:

- If the insurer holds certain long-term assets with cash flows that match the liabilities, this leads to a longer time horizon through the back door by some kind of workaround, one of many required to implement the seemingly purely scientific Solvency II standard.

**Matching Adjustment exemption of Solvency II**

Upward shift of risk-free interest rate to reflect that insurer is not exposed to spread movements due to its long time horizon.

Introducing a scaling factor to offset negative equities with a margin \(d\) above zero

A smaller area on the left side is sufficient to neutralize the normal distribution distances on the right side.

<table>
<thead>
<tr>
<th>area</th>
<th>Left Gaussian bell curve</th>
<th>right Gaussian bell curve</th>
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<tbody>
<tr>
<td>(e^{-\frac{2dt}{\sigma^2}})</td>
<td>e^{-\frac{2dt}{\sigma^2}}</td>
<td>1</td>
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\(e^{-\frac{2dt}{\sigma^2}}\) can be understood as a scaling factor, since it is independent of \(t\) and depends only on the model parameters.
Patchwork-calculation

**Whole picture:**

\[ \psi(\infty) = e^{\frac{-2tm}{\sigma^2}} \]

**Patchwork-calculation on probabilities:**

- Probability of going out of business at any time
  \[ \psi(\infty) = e^{\frac{-2tm}{\sigma^2}} \]

- Probability of going ruin up to time \( t \)

\[ 1 - \psi(t) \]

- Probability of going out of business later on

\[ 1 - \psi(t) \]

- Probability of being still in business at time \( t \)

\[ 1 - \psi(t) \]

From a presentation by the Italian Insurance Association Ania

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**LTG measures**

**The Matching Adjustment (MA)**

**Eligibility criteria for MA**

**Interest-rate curve – applying MA**

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**Ania**

Dario Focarelli - Solvency II Reviews

16/11/2018
Any exaggeration of liabilities reduces own funds unnecessarily

In particular, this can exaggerate the extent of any real low interest rate problem companies may have and force them to take unnecessary action and/or deviate from the appropriate/optimal asset liability management.